Section 1.1: 2, 4, 7, 8, 25, 27, 29, 30, 31, 35, 45, 47, 48, 63

2. If

$$f(x) = \frac{x^2 - x}{x - 1} \qquad \text{and} \qquad g(x) = x$$

is it true that $f = g$?

Though

$$f(x) = \frac{x^2 - x}{x-1} = \frac{x(x-1)}{x-1} = x \text{ if } x \neq 1,$$

we have that

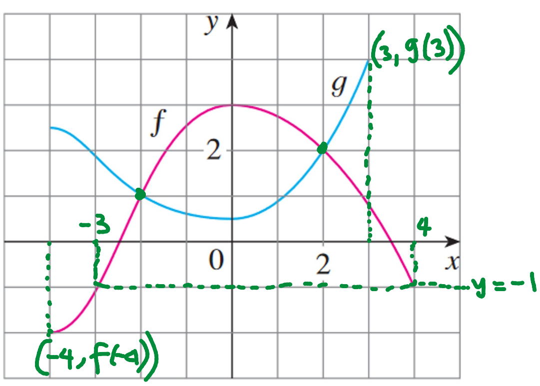
$$f(1) = \frac{1^2 - 1}{1 - 1} = \frac{0}{0} \text{ (undefined)}$$

So $f(x) = g(x)$ for all $x \neq 1$ but $f(1) \neq g(1)$.

Hence,

$$f \neq g.$$

4. The graphs of f and g are given.



(a) State the values of $f(-4)$ and $g(3)$.

$f(-4) = -2$ and $g(3) = 4$

(b) For what values of x is $f(x) = g(x)$?

$f(-2) = 1 = g(-2)$ and $f(2) = 2 = g(2)$

So $f(x) = g(x)$ at $x = -2$ and $x = 2$.

(c) Estimate the solution of the equation $f(x) = -1$.

$f(x) = -1 \Rightarrow x = -3$ and $x = 4$

(d) On what interval is f decreasing?

f is decreasing on $(0, 4)$

(e) State the domain and range of f.

$$\text{Domain}(f) = \{x \in \mathbb{R} : f(x) \text{ is defined}\}$$
$$= [-4, 4]$$

$$\text{Range}(f) = \{y \in \mathbb{R} : f(x) = y \text{ for some } x\}$$
$$= [-2, 3]$$

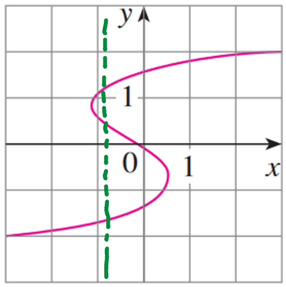
(f) State the domain and range of g.

$$\text{Domain}(g) = \{x \in \mathbb{R} : g(x) \text{ is defined}\}$$
$$= [-4, 3].$$

$$\text{Range}(g) = \{y \in \mathbb{R} : g(x) = y \text{ for some } x\}$$
$$= [1/2, 4].$$

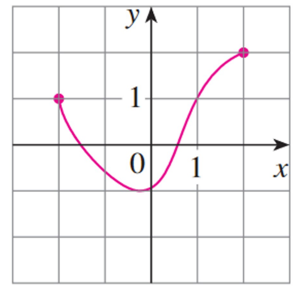
7–10 Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

7.



This is not the graph of a function of x since it fails the vertical line test.

8.



This is the graph of a function since it passes the vertical line test.

Domain is $[-2, 2]$ and

Range is $[-1, 2]$.

25. If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a+1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a+h)$.

$f(2) = 3(2)^2 - 2 + 2 = 12.$

$f(-2) = 3(-2)^2 - (-2) + 2 = 16.$

$f(a) = 3(a)^2 - a + 2 = 3a^2 - a + 2$

$f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2$

$$
\begin{aligned}
f(a+1) &= 3(a+1)^2 - (a+1) + 2 \\
&= 3(a^2 + 2a + 1) - a - 1 + 2 \\
&= 3a^2 + 6a + 3 - a - 1 + 2 \\
&= 3a^2 + 5a + 4
\end{aligned}
$$

$$2f(a) = 2\left[3(a)^2 - a + 2\right]$$

$$= 6a^2 - 2a + 4$$

$$f(2a) = 3(2a)^2 - (2a) + 2$$

$$= 3(4a^2) - 2a + 2$$

$$= 12a^2 - 2a + 2$$

$$f(a^2) = 3(a^2)^2 - (a^2) + 2$$

$$= 3(a^4) - a^2 + 2$$

$$= 3a^4 - a^2 + 2$$

$$[f(a)]^2 = \left[3a^2 - a + 2\right]^2$$ from $f(a)$

$$= (3a^2 - a + 2)(3a^2 - a + 2)$$

$$= 3a^2(3a^2 - a + 2) - a(3a^2 - a + 2) + 2(3a^2 - a + 2)$$

$$= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4$$

$$= 9a^4 - 6a^3 + 13a^2 - 4a + 4$$

$$f(a+h) = 3(a+h)^2 - (a+h) + 2$$

$$= 3(a^2 + 2ah + h^2) - a - h + 2$$

$$= 3a^2 + 6ah + 3h^2 - a - h + 2$$

27–30 Evaluate the difference quotient for the given function. Simplify your answer.

27. $f(x) = 4 + 3x - x^2, \qquad \dfrac{f(3+h) - f(3)}{h}$

$$f(3+h) = 4 + 3(3+h) - (3+h)^2$$
$$= 4 + 9 + 3h - (9 + 6h + h^2)$$
$$= 4 + 9 + 3h - 9 - 6h - h^2$$
$$= 4 - 3h - h^2$$

$$f(3) = 4 + 3(3) - (3)^2$$
$$= 4 + 9 - 9$$
$$= 4$$

So,

$$\frac{f(3+h) - f(3)}{h} = \frac{(4 - 3h - h^2) - (4)}{h}$$
$$= \frac{4 - 3h - h^2 - 4}{h}$$
$$= \frac{-3h - h^2}{h}$$
$$= \frac{-h(3+h)}{h}$$
$$= -3 - h$$

29. $f(x) = \frac{1}{x}$, $\quad \frac{f(x) - f(a)}{x - a}$

$f(x) = \frac{1}{x}$ and $f(a) = \frac{1}{a}$.

So,

$$f(x) - f(a) = \frac{1}{x} - \frac{1}{a}$$

$$= \frac{a - x}{ax}$$

Thus,

$$\frac{f(x) - f(a)}{x - a} = \frac{a - x}{ax} \div \frac{x - a}{1}$$

$$= \frac{a - x}{ax} \times \frac{1}{x - a}$$

$$= \frac{-(x - a)}{ax(x - a)}$$

$$= -\frac{1}{ax}$$

30. $f(x) = \frac{x + 3}{x + 1}$, $\quad \frac{f(x) - f(1)}{x - 1}$

$f(x) = \frac{x+3}{x+1}$ and $f(1) = \frac{1+3}{1+1} = \frac{4}{2} = 2$.

So,

$$f(x) - f(1) = \frac{x+3}{x+1} - 2$$

$$= \frac{x+3-2(x+1)}{x+1}$$

$$= \frac{x+3-2x-2}{x+1}$$

$$= \frac{1-x}{x+1}$$

Thus,

$$\frac{f(x)-f(1)}{x-1} = \frac{1-x}{x+1} \div \frac{x-1}{1}$$

$$= \frac{1-x}{x+1} \times \frac{1}{x-1}$$

$$= \frac{-\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$$

$$= -\frac{1}{x+1}$$

31–37 Find the domain of the function.

31. $f(x) = \dfrac{x+4}{x^2-9}$

$$f(x) = \frac{x+4}{x^2-9} = \frac{x+4}{(x-3)(x+3)}$$

$$\text{Domain}(f) = \{x \in \mathbb{R} : (x-3)(x+3) \neq 0\}$$

$$= \{x \in \mathbb{R} : x \neq 3 \text{ and } x \neq -3\}$$

$$= (-\infty, -3) \cup \underline{\underline{(-3, 3)}} \cup (3, \infty)$$

35. $h(x) = \dfrac{1}{\sqrt[4]{x^2 - 5x}}$

$$h(x) = \frac{1}{\sqrt[4]{x^2 - 5x}} = \frac{1}{\sqrt[4]{x(x-5)}}$$

$$\text{Domain}(f) = \{x \in \mathbb{R} : x(x-5) > 0\}$$

$$= \{x \in \mathbb{R} : x < 0 \text{ and } x > 5\}$$

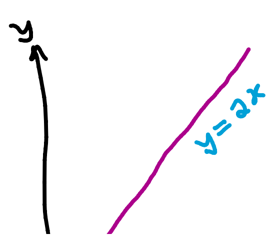
$$= \underline{\underline{(-\infty, 0) \cup (5, \infty)}}$$

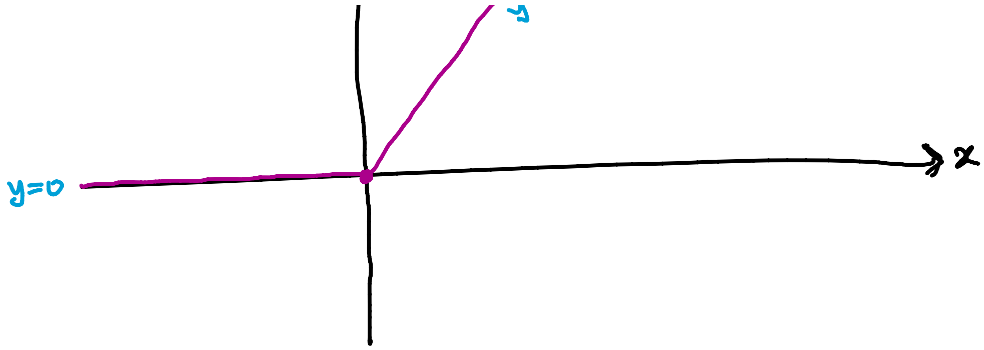
		0		5	
x	$-$		$+$		$+$
$x-5$	$-$		$-$		$+$
$x(x-5)$	$+$		$-$		$+$

45–50 Sketch the graph of the function.

45. $f(x) = x + |x|$

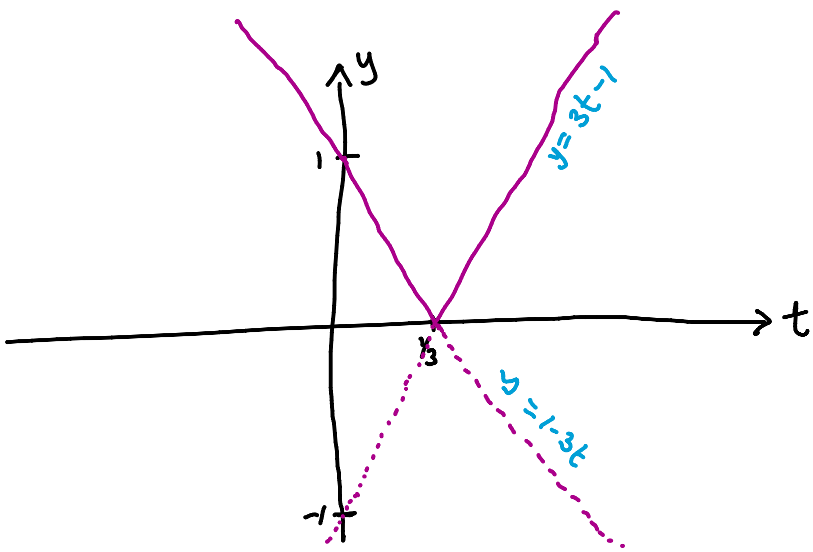
$$f(x) = \begin{cases} x + x & \text{if } x \geq 0 \\ x - x & \text{if } x < 0 \end{cases} = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$





47. $g(t) = |1 - 3t|$

$$g(t) = \begin{cases} 1-3t & \text{if } 1-3t \geq 0 \\ -(1-3t) & \text{if } 1-3t < 0 \end{cases} = \begin{cases} 1-3t & \text{if } t \leq 1/3 \\ 3t-1 & \text{if } t > 1/3 \end{cases}$$



48. $h(t) = |t| + |t + 1|$

$$|t| = \begin{cases} t, & t \geq 0 \\ -t, & t < 0 \end{cases} \quad \text{and} \quad |t+1| = \begin{cases} t+1, & t+1 \geq 0 \\ -t-1, & t+1 < 0 \end{cases}$$

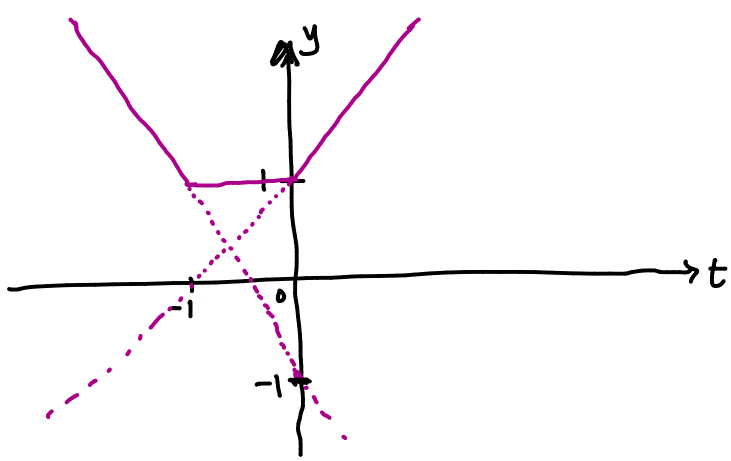
$$= \begin{cases} t, & t \geq 0 \\ -t, & t < 0 \end{cases} \quad \text{and} \quad |t+1| = \begin{cases} t+1, & t \geq -1 \\ -t-1, & t < -1 \end{cases}$$

So,

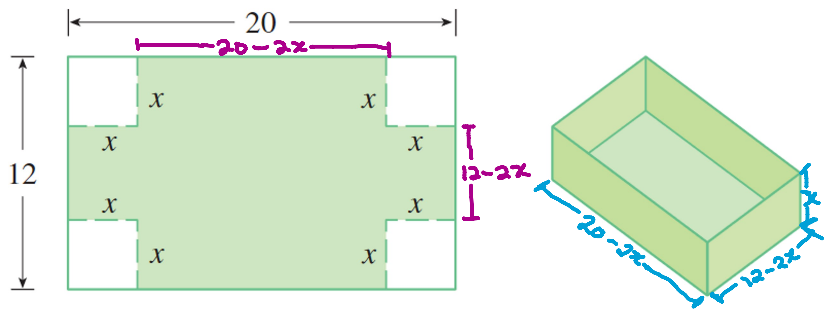
$$h(t) = |t| + |t+1|$$

$$= \begin{cases} t + (t+1) & \text{if } t \geq 0 \\ -t + (t+1) & \text{if } -1 \leq t < 0 \\ -t - t - 1 & \text{if } t < -1 \end{cases}$$

$$= \begin{cases} 2t + 1 & \text{if } t \geq 0 \\ 1 & \text{if } -1 \leq t < 0 \\ -2t - 1 & \text{if } t < -1 \end{cases}$$



63. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.



$$\text{Volume} = Lwh$$
$$= (20-2x)(12-2x)x$$
$$= (20-2x)(12x-2x^2)$$
$$= 20(12x-2x^2) - 2x(12x-2x^2)$$
$$= 240x - 40x^2 - 24x^2 + 4x^3$$
$$= 240x - 64x^2 + 4x^3$$

But

$$\left.\begin{array}{l} x > 0 \text{ and} \\ L = 20-2x > 0 \text{ and} \\ W = 12-2x > 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x > 0 \text{ and} \\ x < 10 \text{ and} \\ x < 6 \end{array}\right\} \Rightarrow 0 < x < 6$$

Hence, the volume of the box as a function of x is

$$V(x) = 240x - 64x^2 + 4x^3$$

with domain $(0, 10)$.