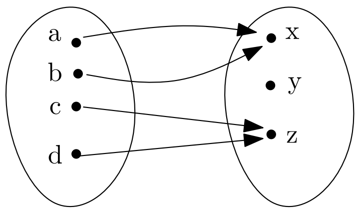
Lecture Note 1 (Ref. text book page 10) 08/17/2021

1.1 Four Ways to Represent a Function

A **function**, call it f, is a rule that assigns to each element x in a set D **exactly** one element, called $f(x)$, in a set E.



i.e. $f(a) = f(b) = x$

$f(c) = f(d) = z$

- For this course, we always consider that the sets A and B are the subsets of real numbers i.e. $A \subseteq \mathbb{R}$ and $B \subseteq \mathbb{R}$.
- The symbol $f(x)$ is read "f of x" or "f at x" and is called the value of f at x, or the **image** of x under f.

The set A is called the **domain** of the function. The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain, that is the

$$\text{range of } f = \{f(x) | x \in D\}$$

There are four possible ways to represent a function

- Verbally: By Description in words
- Numerically: By table of values
- Visually: By graph
- Algebraically: By an explicit formula

Example 1

Let $f(x) = 3x + 2$. Evaluate $\dfrac{f(a+h) - f(a)}{h}$, $h \neq 0$

Example 2

Evaluate the piecewise defined functions at the indicated values.

$$f(x) = \begin{cases} 3x & \text{if} \quad x < 0 \\ x + 1 & \text{if} \quad 0 \leq x \leq 2 \\ (x-2)^2 & \text{if} \quad x > 2 \end{cases} \qquad f(-5),\ f(0),\ f(1),\ f(2),\ f(5)$$

The Domain of a Function

The domain of a function may be stated explicitly, for example if we write

$$f(x) = x^3 - 1, \quad 2 \leq x \leq 15$$

then the domain of the function is $2 \leq x \leq 15$.

If the function is given by a mathematical formula and the domain is not stated explicitly, then *the domain of the function is the set of all real numbers for which the expression is defined as a real number.*

Example 3

Find the domain of each function.

(a) $f(x) = \dfrac{x+2}{x^2-1}$ (b) $f(x) = \sqrt{2x-5}$ (c) $f(x) = \dfrac{3}{\sqrt{x-4}}$

The graph of a function is a curve in the xy-plane. But question arises: Which curves in the xy-plane are graphs of functions? This is answered by the following test.

The Vertical Line Test: A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

Even and Odd Functions

Let f be a function.

- f is **even** if $f(-x) = f(x)$ for all x in the domain of f.
  The graph of an even function is symmetric with respect to the y-axis.
- f is **odd** if $f(-x) = -f(x)$ for all x in the domain of f.
  The graph of an odd function is symmetric with respect to the origin.

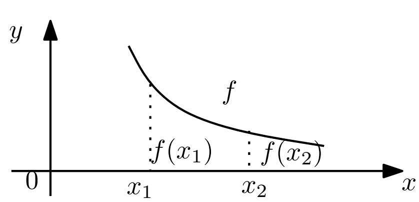
Example 4 Determine whether the function f is even, odd, or neither.

(a) $f(x) = 4x^4 + 2x^2 + 1$ (b) $f(x) = x^3 + x$ (c) $f(x) = x^2 + x - 1$

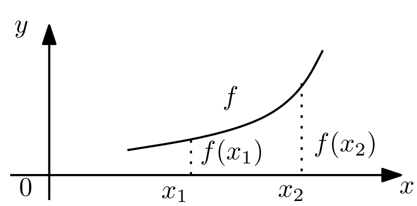
Definition of Increasing and Decreasing Functions

- A function f is **increasing** on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
- A function f is **decreasing** on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.

The definitions mean that the function f is said to be *increasing* when its graph rises and *decreasing* when its graph falls.



f is decreasing



f is increasing