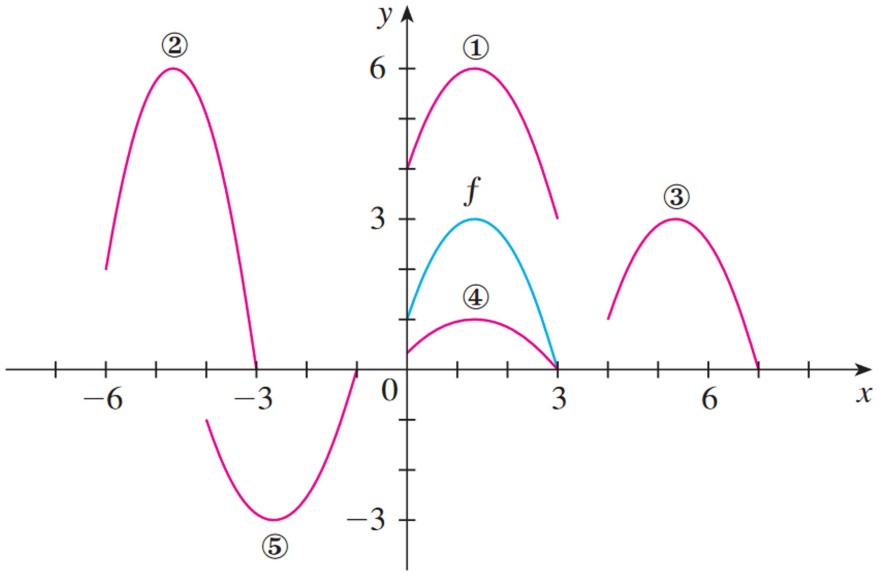
3. The graph of $y = f(x)$ is given. Match each equation with its graph and give reasons for your choices.

(a) $y = f(x - 4)$ (b) $y = f(x) + 3$
(c) $y = \frac{1}{3}f(x)$ (d) $y = -f(x + 4)$
(e) $y = 2f(x + 6)$



(a) [graph 3] since $y = f(x-4)$ is a horizontal shift to the right of $y = f(x)$ by 4.

(b) [graph 1] since $y = f(x) + 3$ is an upward vertical shift of $y = f(x)$ by 3.

(c) [graph 4] since $y = \frac{1}{3}f(x)$ shrinks $y = f(x)$ vertically by a factor of 3.

(d) [graph 5] since $y = -f(x+4)$ is a horizontal shift to the left by 4 and a reflection about x-axis.

(e) [graph 2] since $y = 2f(x+6)$ is a horizontal shift to the left by 6 and a vertical stretch by a factor of 2.

31–32 Find (a) $f + g$, (b) $f - g$, (c) fg, and (d) f/g and state their domains.

31. $f(x) = x^3 + 2x^2, \quad g(x) = 3x^2 - 1$

(a) $(f+g)(x) = f(x) + g(x)$

$= (x^3 + 2x^2) + (3x^2 - 1)$

$= x^3 + 5x^2 - 1$

Dom(f) = all real numbers $= (-\infty, \infty)$

Dom(g) = all real numbers $= (-\infty, \infty)$

So, Dom(f+g) $= (-\infty, \infty) \cap (-\infty, \infty)$

$= (-\infty, \infty)$

(b) $(f-g)(x) = f(x) - g(x)$

$= (x^3 + 2x^2) - (3x^2 - 1)$

$= x^3 + 2x^2 - 3x^2 + 1$

$= x^3 - x^2 + 1$

Dom(f-g) = Dom(f) ∩ Dom(g)

$= (-\infty, \infty) \cap (-\infty, \infty)$

$= (-\infty, \infty)$

(c) $(fg)(x) = f(x)\,g(x)$

$= (x^3 + 2x^2)(3x^2 - 1)$

$= x^3(3x^2 - 1) + 2x^2(3x^2 - 1)$

$= 3x^5 - x^3 + 6x^4 + 2x^2$

$= 3x^5 + 6x^4 - x^3 + 2x^2$

Dom(fg) = Dom(f) ∩ Dom(g)

$= (-\infty, \infty) \cap (-\infty, \infty)$

$= (-\infty, \infty)$

(d) $\left(\frac{f}{g}\right)(x) = \frac{x^3 + 2x^2}{\quad}$ $\quad g(x) = 0 \Rightarrow 3x^2 - 1 = 0 \Rightarrow x = \pm \frac{1}{\sqrt{3}}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{x^3+2x^2}{3x^2-1}$

$g(x)=0 \Rightarrow 3x^2-1=0 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

So $g(x)\neq 0$ on $(-\infty,-\frac{1}{\sqrt{3}})\cup(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})\cup(\frac{1}{\sqrt{3}},\infty)$

Hence,

$\text{Dom}\left(\frac{f}{g}\right) = \text{Dom}(f)\cap\text{Dom}(g)\cap\left((-\infty,-\frac{1}{\sqrt{2}})\cup(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{3}})\cup(\frac{1}{\sqrt{3}},\infty)\right)$

$= (-\infty,\infty)\cap(-\infty,\infty)\cap\left((-\infty,-\frac{1}{\sqrt{3}})\cup(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})\cup(\frac{1}{\sqrt{3}},\infty)\right)$

$= (-\infty,-\frac{1}{\sqrt{3}})\cup(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})\cup(\frac{1}{\sqrt{3}},\infty)$

33–38 Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$ and their domains.

33. $f(x) = 3x + 5, \quad g(x) = x^2 + x$

(a) $(f\circ g)(x) = f(g(x))$

$= f(x^2+x)$

$= 3(x^2+x)+5$

$= 3x^2+3x+5$

$\text{Dom}(f) = (-\infty,\infty)$, $\text{Dom}(g) = (-\infty,\infty)$

$\underbrace{\text{Dom}(f\circ g)}_{\text{as a composition}} = \underbrace{\text{Dom}(f\circ g)}_{\text{as a new function}} \cap \text{Dom}(g)$

$= (-\infty,\infty)\cap(-\infty,\infty)$

$= (-\infty,\infty)$

(b) $(g\circ f)(x) = g(f(x))$

$= g(3x+5)$

$= (3x+5)^2+(3x+5)$

$\underbrace{\text{Dom}(g\circ f)}_{\text{as composition}} = \underbrace{\text{Dom}(g\circ f)}_{\text{as a new function}} \cap \text{Dom}(f)$

$$= (3x+5)^2 + (3x+5)$$

$$= 9x^2 + 30x + 25 + 3x + 5$$

$$= 9x^2 + 33x + 30$$

function

$$= (-\infty, \infty) \cap (-\infty, \infty)$$

$$= (-\infty, \infty)$$

(c) $(f \circ f)(x) = f(f(x))$

$$= f(3x+5)$$

$$= 3(3x+5) + 5$$

$$= 9x + 15 + 5$$

$$= 9x + 20$$

Just like in (a) and (b),

$$\text{Dom}(f \circ f) = \text{Dom}(f \circ f) \cap \text{Dom}(f)$$

$$= (-\infty, \infty) \cap (-\infty, \infty)$$

$$= (-\infty, \infty)$$

(d) $(g \circ g)(x) = g(g(x))$

$$= g(x^2 + x)$$

$$= (x^2+x)^2 + (x^2+x)$$

$$= x^4 + 2x^3 + x^2 + x^2 + x$$

$$= x^4 + 2x^3 + 2x^2 + x$$

Just like in (c),

$$\text{Dom}(g \circ g) = \text{Dom}(g \circ g) \cap \text{Dom}(g)$$

$$= (-\infty, \infty) \cap (-\infty, \infty)$$

$$= (-\infty, \infty)$$

39–42 Find $f \circ g \circ h$.

39. $f(x) = 3x - 2, \quad g(x) = \sin x, \quad h(x) = x^2$

$$(f \circ g \circ h)(x) = f(g(h(x)))$$

$$= f(g(x^2))$$

$$= f(\sin(x^2))$$

$$= 3(\sin(x^2)) - 2$$

$$= 3\sin(x^2) - 2$$