Reminder: Basic Functions we're studying

* Power function (eg. $f(x) = x^5$)
* Root function (eg. $f(x) = \sqrt{x}$
* Polynomial (eg. $f(x) = 2 + 3x + 4x^2$)
* Rational function (e.g. $f(x) = \frac{1+x}{x^2+1}$)
* Algebraic function (eg. $f(x) = \frac{\sqrt{2x+1}}{\sqrt[3]{x} + 1}$)
* Trigonometric function (eg. $f(x) = \tan x - \cos x$)
* Exponential function (eg. $f(x) = \pi^x$)
* Logarithmic function (eg. $f(x) = \log_2 x$)

Sec 1.4 # 4

4 a

$$\frac{x^{2n}\cdot x^{3n-1}}{x^{n+2}} = \frac{x^{2n+(3n-1)}}{x^{n+2}}$$

$$= \frac{x^{2n+3n-1}}{x^{n+2}}$$

$$= x^{5n-1-(n+2)}$$

$$= x^{4n-3}$$

b

$$\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \frac{(a\,b^{1/2})^{1/2}}{(ab)^{1/3}} = \frac{a^{1/2}\,b^{1/4}}{a^{1/3}\,b^{1/3}}$$

$$= a^{\frac{1}{2}-\frac{1}{3}}\cdot b^{\frac{1}{4}-\frac{1}{3}} = a^{1/6}\,b^{-1/12}$$

19 (a) $f(x) = \dfrac{1-e^{x^2}}{1-e^{1-x^2}}$. Denominator is zero when

$1-e^{1-x^2} = 0 \iff e^{1-x^2} = 1 \iff \ln e^{1-x^2} = \ln 1$

$\iff (1-x^2)\ln e = 0 \iff (1-x)(1+x) = 0$

$\iff x = -1, 1$

So

$D(f) = \{x \in \mathbb{R} : x \neq -1 \text{ and } x \neq 1\}$

$= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

(b) $f(x) = \dfrac{1+x}{e^{\cos x}}$. Denominator $e^{\cos x} \neq 0$ for any x.

So

$D(f) = \{x \in \mathbb{R}\}$

$= (-\infty, \infty)$.

21 $(1,6)$ on the graph of $f(x) = cb^x$

$\Longrightarrow 6 = cb^1 \Longrightarrow c = \frac{6}{b}$, (Remeber $b>0$)

Similarly, $(3,24)$ on the graph

$\Longrightarrow 24 = cb^3 \Longrightarrow 24 = \frac{6}{b} \cdot b^3 = 6b^2$

$\Longrightarrow \frac{24}{6} = b^2 \Longrightarrow 4 = b^2$

$\Longrightarrow b = 2$ since $b>0$

$\Longrightarrow c = \frac{6}{b} = \frac{6}{2} = 3$.

Hence $f(x) = 3 \cdot 2^x$ is the required function.