Lecture Note 3 (Ref. text book page 45)

08/20/2021

1.4 Exponential Functions

The function $f(x) = 2^x$ is called an exponential function because the variable, x, is the exponent. It should not be confused with the power function $g(x) = x^2$, in which the variable is the base.

In general, an exponential function is a function of the form

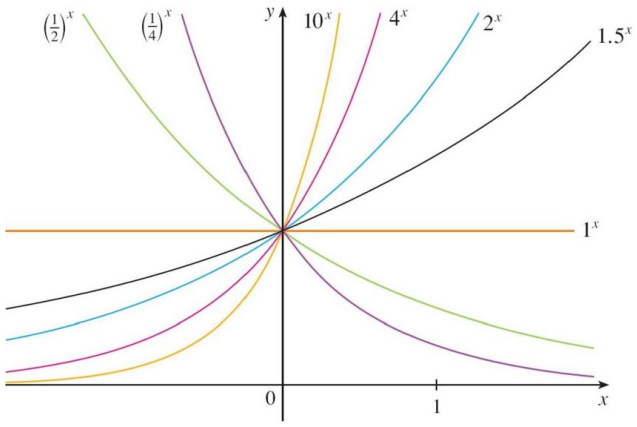
$$f(x) = b^x$$

where b is a positive constant.

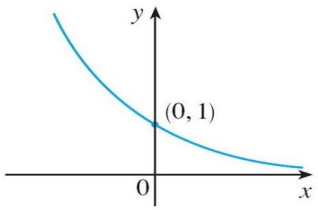
We call the function

$$f(x) = e^x$$

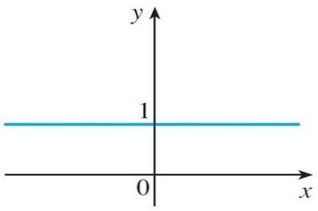
the natural exponential function, where e denotes the Euler number. An approximation of the number e to five decimal is 2.71828



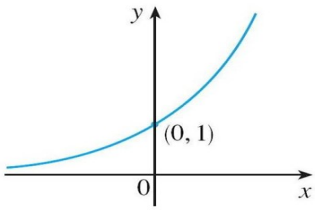
Note that all of these graphs pass through the same point $(0, 1)$ because $b^0 = 1$ for $b \neq 0$. If $0 < b < 1$, the exponential function decreases; if $b = 1$, it is a constant; and if $b > 1$, it increases. These three cases are illustrated in the next Figure .



(a) $y = b^x$, $0 < b < 1$



(b) $y = b^x$, $b = 1$



(c) $y = b^x$, $b > 1$

Observe that if $b \neq 1$, then the exponential function $y = b^x$ has domain $\mathbb{R}$ and range $(0, +\infty)$. Notice also that the graph of $y = (1/b)^x$ is just the reflection of the graph of $y = b^x$ about the y-axis.

Example 1 Sketch the graph of $y = 3 - 2^x$ and determine its domain and range.

Laws of Exponents If a and b are positive numbers and x and y are any real numbers, then

1. $b^{x+y} = b^x b^y$ **2.** $b^{x-y} = \dfrac{b^x}{b^y}$ **3.** $(b^x)^y = b^{xy}$ **4.** $(ab)^x = a^x b^x$

Example 2. Use the Law of Exponents to rewrite and simplify the expressions

$$(a)\ \frac{4^{-3}}{2^{-8}}, \qquad (b)\ \frac{1}{\sqrt[2]{x^4}},$$

$$(c) b^8(2b)^4 \qquad (d) \frac{(6y^3)^4}{2y^5},$$

Example 3. The half-life of strontium-90, ^{90}Sr, is 25 years. This means that half of any given quantity of ^{90}Sr will disintegrate in 25 years.
(a) If a sample of ^{90}Sr has a mass of 24 mg, find an expression for the mass m(t) that remains after t years.
(b) Find the mass remaining after 40 years, correct to the nearest milligram.