

- * Power function (eg. $f(x) = x^5$)
- * Root function (eg. $f(x) = \sqrt{x}$)
- * Polynomial (eg. $f(x) = 2 + 3x + 4x^2$)
- * Rational function (eg. $f(x) = \frac{1+x}{x^2+1}$)
- * Algebraic function (eg. $f(x) = \frac{\sqrt{2x+1}}{\sqrt{x}+1}$)
- * Trigonometric function (eg. $f(x) = \tan x - \cos x$)
- * Exponential function (eg. $f(x) = \pi^x$)
- * Logarithmic function (eg. $f(x) = \log_2 x$)

You should be able to recognize these functions and their graphs

$$\textcircled{a} \lim_{x \rightarrow -7} f(x) = -\infty$$

$$\textcircled{b} \lim_{x \rightarrow -3} f(x) = \infty$$

$$\textcircled{c} \lim_{x \rightarrow 0} f(x) = \infty$$

$$\textcircled{d} \lim_{x \rightarrow 6^-} f(x) = -\infty$$

$$\textcircled{e} \lim_{x \rightarrow 6^+} f(x) = \infty$$

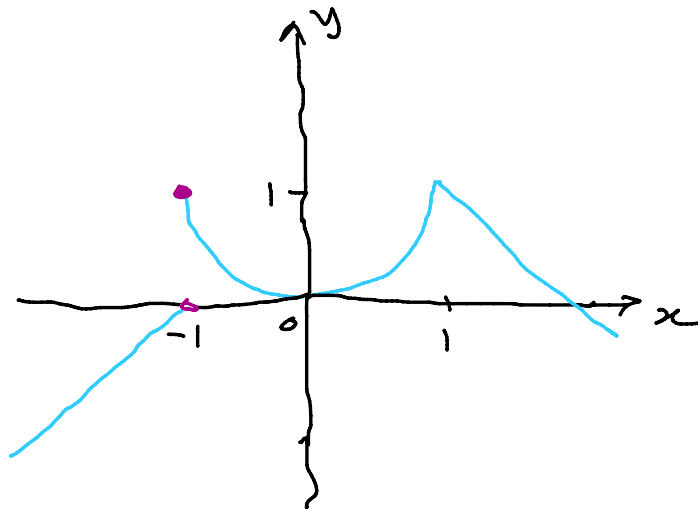
\textcircled{f} Equations of vertical asymptotes are
 $x = -7, x = -3, x = 0, x = 6$

$$f(x) = \begin{cases} 1+x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 1 \\ 2-x & \text{if } x \geq 1 \end{cases}$$

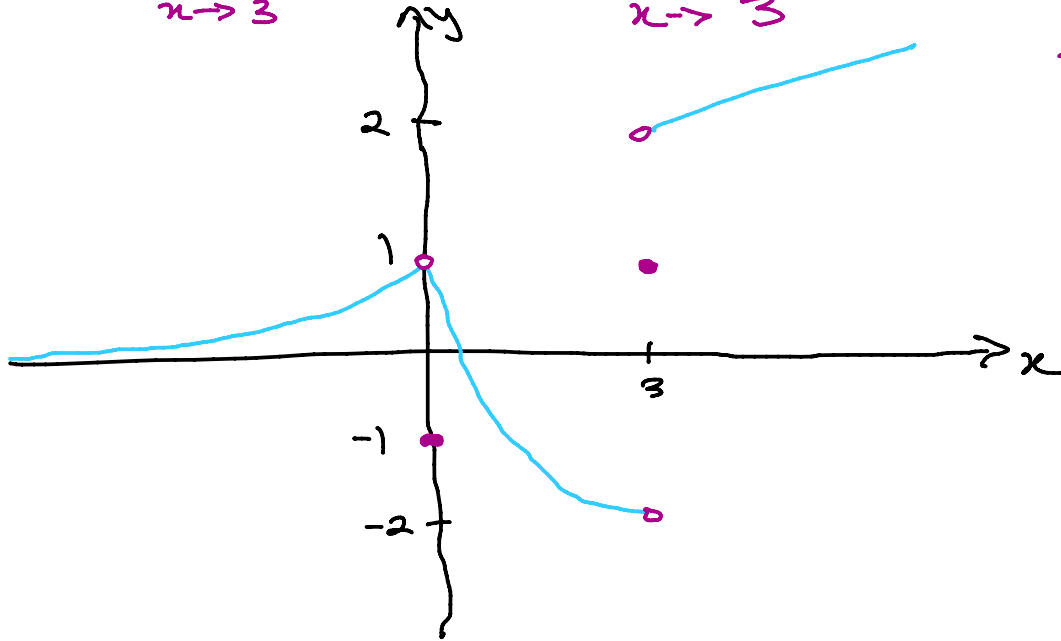
$\lim_{x \rightarrow a} f(x)$ exists everywhere
on \mathbb{R} except at
 $a = -1$

where

$$\lim_{x \rightarrow a^-} f(x) = 0 \quad \text{and} \quad \lim_{x \rightarrow a^+} f(x) = 1$$



$\lim_{x \rightarrow 0} f(x) = 1$, $\lim_{x \rightarrow 3^-} f(x) = -2$, $\lim_{x \rightarrow 3^+} f(x) = 2$, $f(0) = -1$
 $f(3) = 1$



①7 Repeat similarly.

$$\lim_{x \rightarrow 5^+} \frac{x+1}{x-5} = \infty \text{ since } x+1 \text{ is positive and } x-5 > 0$$

as $x \rightarrow 5^+$

$$\textcircled{35} \lim_{x \rightarrow 3^+} \ln(x^2-9) = \lim_{t \rightarrow 0^+} \ln t = -\infty \text{ since } t = x^2-9$$

approaches 0 as x
approaches 3^+

$$\textcircled{41} \lim_{x \rightarrow 2^+} \frac{x^2-2x-8}{x^2-5x+6} = \lim_{x \rightarrow 2^+} \frac{(x-4)(x+2)}{(x-3)(x-2)}$$

$= \infty$ since $(x-4)(x+2) < 0$ as $x \rightarrow 2^+$
and $(x-3)(x-2) < 0$ as $x \rightarrow 2^+$

$$\textcircled{42} \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \ln x \right) = \lim_{x \rightarrow 0^+} \frac{1}{x} - \lim_{x \rightarrow 0^+} \ln x$$

$$= \infty - (-\infty)$$

$$= \infty + \infty$$

$$= \infty$$

Notice that:

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x} + \ln x \right) = \infty \text{ since } \frac{1}{x} \rightarrow \infty \text{ and } \ln x \rightarrow -\infty$$

as $x \rightarrow 0^+$ but $\frac{1}{x}$ grows
faster than $\ln x$ decreases
as $x \rightarrow 0^+$