Sect 2.3 # 4, 6

4 $\lim_{x \to -1} (x^4 - 3x)(x^2 + 5x + 3)$

$= \lim_{x \to -1} (x^4 - 3x) \lim_{x \to -1} (x^2 + 5x + 3)$

$= \left((-1)^4 - 3(-1)\right)\left((-1)^2 + 5(-1) + 3\right)$

$= (1+3)(1-5+3) = 4(-1)$

$= \underline{\underline{-4}}$

6 $\lim_{u \to -2} \sqrt{u^4 + 3u + 6} = \sqrt{\lim_{x \to -2} (u^4 + 3u + 6)}$

$= \sqrt{(-2)^4 + 3(-2) + 6}$

$= \sqrt{16 - 6 + 6}$

$= \sqrt{16}$

$= 4$ //

Sect 2.3 #12, 22, 26

$$\lim_{x\to -3} \frac{x^2+3x}{x^2-x-12} = \lim_{x\to -3} \frac{x(x+3)}{(x-4)(x+3)}$$

$$= \lim_{x\to -3} \frac{x}{x-4}$$

$$= \frac{-3}{-3-4}$$

$$= \frac{3}{7}$$

(22) $$\lim_{u\to 2} \frac{\sqrt{4u+1}-3}{u-2}$$

$$= \lim_{u\to 2} \frac{(\sqrt{4u+1}-3)(\sqrt{4u+1}+3)}{(u-2)(\sqrt{4u+1}+3)}$$

$$= \lim_{u\to 2} \frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3)}$$

$$= \lim_{x\to 2} \frac{4(u-2)}{(u-2)(\sqrt{4u+1}+3)}$$

$$= \lim \frac{4}{\quad}$$

$x \to 2 \qquad \sqrt{4u+1} + 3$

$$= \frac{4}{\sqrt{4(2)+1} + 3} = \frac{4}{\sqrt{9} + 3} = \frac{4}{6} = \frac{2}{3}$$

26 $$\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right)$$

$$= \lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t(t+1)} \right)$$

$$= \lim_{t \to 0} \left(\frac{t+1-1}{t(t+1)} \right)$$

$$= \lim_{t \to 0} \left(\frac{t}{t(t+1)} \right)$$

$$= \lim_{t \to 0} \frac{1}{t+1} = \frac{1}{0+1} = \frac{1}{1} = 1$$

Sect 2.3 # 38

$2x \le g(x) \le x^4 - x^2 + 2$, find $\lim_{x \to 1} g(x)$

$$\lim_{x \to 1} 2x = 2(1) = 2$$

and

$$\lim_{x \to 1} (x^4 - x^2 + 2) = 1^4 - 1^2 + 2 = 2$$

By Squeeze Theorem,

$$\lim_{x \to 1} (2x) \le \lim_{x \to 1} g(x) \le \lim_{x \to 1} (x^4 - x^2 + 2)$$

$$\Rightarrow \quad 2 \le \lim_{x \to 1} g(x) \le 2$$

So $\lim_{x \to 1} g(x) = 2$ //

Sect 2.3 # 45, 46

45 $\lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{|x|}\right) = \lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{-x}\right)$

$= \lim_{x\to 0^-}\left(\frac{2}{x}\right) = -\infty$

Hence, the limit does not exist.

46 $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{|x|}\right) = \lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x}\right)$

$= \lim_{x\to 0^+} 0$

$= 0$

Notice that combining 45 and 46, we conclude that

$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{|x|}\right)$ does not exist.