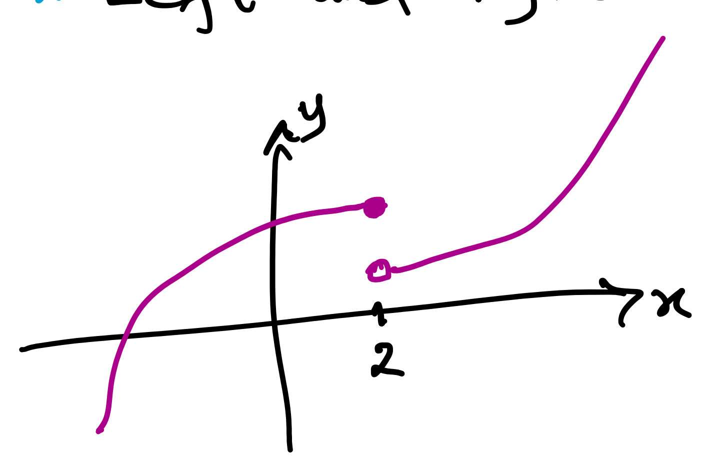
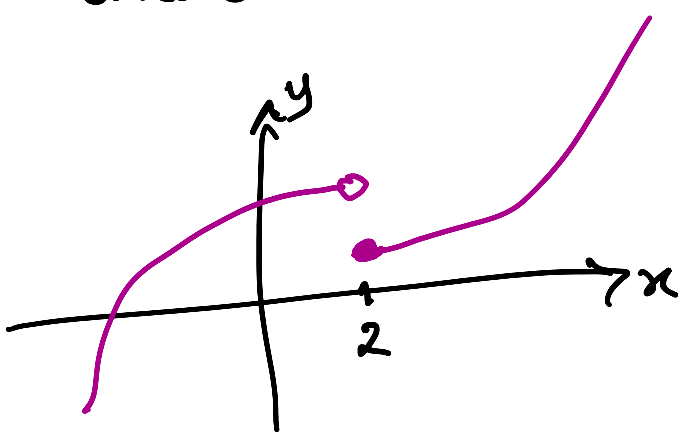
2.5 Continuity

Thursday, August 26, 2021 12:29 PM

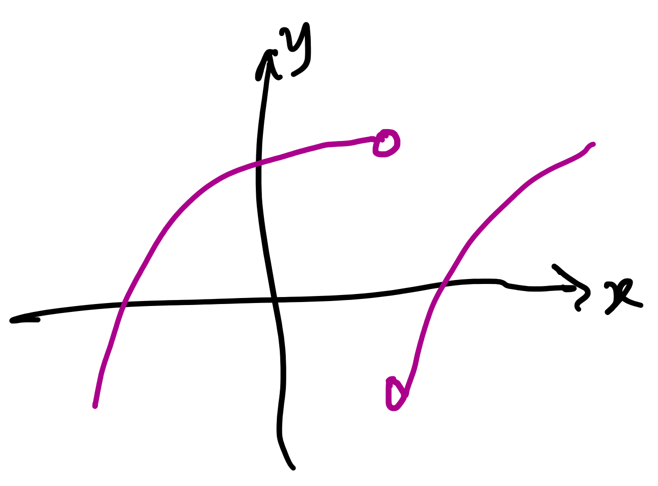
* Left and right continuities are included as continuities. So



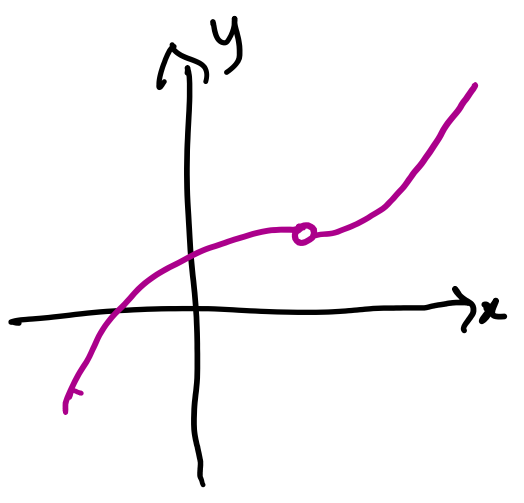


are continuous at $x = 2$

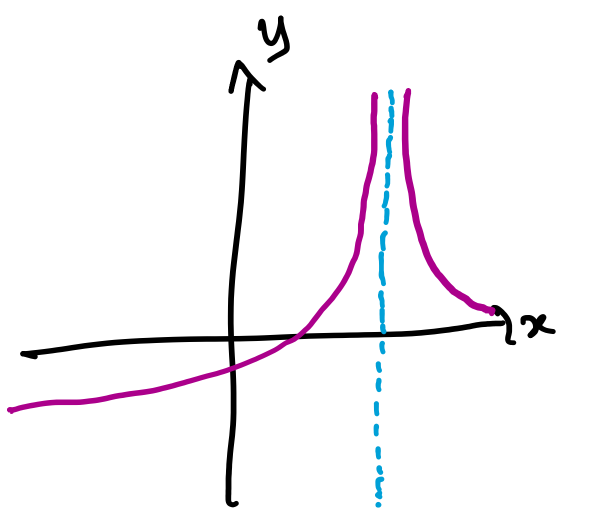
* Types of discontinuities:
  1. Jump
  2. Removable
  3. Infinite



Jump discontinuity



Removable discontinuity



Infinite discontinuity

* All the basic functions we have seen: polynomial, rational, exponential, logarithmic, root, trigonometric and inverse functions are continuous in their domains.

* Thus, think of many of the weird piecewise functions which are not continuous as exceptions rather than the rule. The following examples illustrate this.

* Use continuity to evaluate the limit:

(35) $\lim_{x\to 2} x\sqrt{20-x^2} = 2\sqrt{20-2^2}$
$= 2\sqrt{16}$
$= 2(4)$
$= 8$

	$x<-\sqrt{20}$	$-\sqrt{20}<x<\sqrt{20}$	$x>\sqrt{20}$
$\sqrt{20}-x$	+	+	−
$\sqrt{20}+x$	−	+	+
$\sqrt{20-x^2}$	−	+	−

since $\mathrm{Dom}\left(x\sqrt{20-x^2}\right) = \{x\in\mathbb{R}: 20-x^2\geq 0\}$
$= [-\sqrt{20}, \sqrt{20}]$ which contains $x=2$.

(37) $\lim_{x\to 1} \ln\left(\frac{5-x^2}{1+x}\right) = \ln\left(\frac{5-1^2}{1+1}\right)$
$= \ln\left(\frac{4}{2}\right)$
$= \ln 2$

	$x<-\sqrt{5}$	$-\sqrt{5}<x<-1$	$-1<x<\sqrt{5}$	$x>\sqrt{5}$
$\sqrt{5}-x$	+	+	+	−
$\sqrt{5}+x$	−	+	+	+
$1+x$	−	−	+	+
$\frac{5-x^2}{1+x}$	+	−	+	−

since $\mathrm{Dom}\left(\ln\left(\frac{5-x^2}{1+x}\right)\right) = \{x\in\mathbb{R}: \frac{5-x^2}{1+x} > 0\}$
$= (-\infty,-\sqrt{5})\cup(-1,\sqrt{5})$ which contains $x=1$

(38) $\lim_{x\to 4} 3^{\sqrt{x^2-2x-4}} = 3^{\sqrt{4^2-2(4)-4}}$
$= 3^{\sqrt{16-8-4}}$
$= 3^{\sqrt{4}}$
$= 3^2$
$= 9$

	$x<1-\sqrt{5}$	$1-\sqrt{5}<x<1+\sqrt{5}$	$x>1+\sqrt{5}$
$x-1-\sqrt{5}$	−	−	+
$x-1+\sqrt{5}$	−	+	+
x^2-2x-4	+	−	+

since $\mathrm{Dom}\left(3^{\sqrt{x^2-2x-4}}\right) = \{x\in\mathbb{R}: x^2-2x-4\geq 0\}$
$(x-1-\sqrt{5})(x-1+\sqrt{5})$

$= (-\infty, 1-\sqrt{5}]\cup[1+\sqrt{5},\infty)$ which contains $x=4$.

All of these functions behave well in a very close neighborhood of the points we are taking limits at $x = 2, 1, 4$.