(11) $f(x) = (x+2x^3)^4$, $a=-1$

$$f(-1) = (-1+2(-1)^3)^4 = (-1-2)^4 = (-3)^4 = 81$$

and

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} (x+2x^3)^4 = \left(\lim_{x\to -1} x + 2\lim_{x\to -1} x^3\right)^4$$

$$= (-1+2(-1)^3)^4$$

$$= (-1-2)^4$$

$$= (-3)^4$$

$$= 81$$

Hence,

$$\lim_{x\to -1} f(x) = 81 = f(-1).$$

So f is continuous at $x=-1$

(15) $f(x) = x + \sqrt{x-4}$, $[4, \infty)$

Notice that

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \left(x + \sqrt{x-4}\right) = 4 = f(4)$$

and for $a>4$,

$$\lim_{x\to a} f(x) = \lim_{x\to a} \left(x+\sqrt{x-4}\right) = \lim_{x\to a} x + \sqrt{\lim_{x\to a} x-4}$$

$$= a + \sqrt{a-4}$$

$= f(a)$

Hence f is continuous from the right at $x = 4$ and continuous on $(4, \infty)$.

(36) $\lim_{x \to \pi} \sin(x + \sin x) = \sin\left(\lim_{x \to \pi} x + \lim_{x \to \pi} \sin x\right)$

$= \sin(\pi + \sin \pi)$

$= \sin(\pi + 0)$ since $\sin \pi = 0$

$= \sin \pi$

$= 0$

(38) $\lim_{x \to 4} 3^{\sqrt{x^2 - 2x - 4}} = 3^{\lim_{x \to 4} \sqrt{x^2 - 2x - 4}}$

$= 3^{\sqrt{\lim_{x \to 4} x^2 - 2 \lim_{x \to 4} x - 4}}$

$= 3^{\sqrt{16 - 2(4) - 4}}$

$= 3^{\sqrt{4}}$

$= 3^2$

$= \underline{\underline{9}}$

(53) $x^4 + x - 3 = 0$, $(1, 2)$

$f(x)$

f is continuous on $[1,2]$ since f is a Polynomial and $f(1) = 1^2+1-3 = -1$, $f(2) = 2^4+2-3 = 15$.
So 0 lies between $f(1)$ and $f(2)$. Thus, by Intermediate Value Theorem, we can find c in $(1,2)$ such that

$$f(c) = 0$$

ie; $x^4+x-3=0$ has a root in the interval $(1,2)$.

(55) $e^x = 3-2x$, $(0,1)$

Let $f(x) = e^x + 2x - 3 = 0$. Then f is continuous on $[0,1]$ and

$$f(0) = e^0 + 2(0) - 3 = -2, \quad f(1) = e^1 + 2(1) - 3 = e-1$$

So 0 lies between $f(0)$ and $f(1)$ and by Intermediate Value Theorem, we can find c in $(0,1)$ such that

$$f(c) = 0$$

ie;

$$e^c + 2c - 3 = 0 \text{ for some } c \text{ in } (0,1)$$

and so

$e^x = 3-2x$ has a root in $(0,1)$.