15 $\lim_{x\to\infty} \frac{3x-2}{2x+1} = \lim_{x\to\infty} \frac{x\left(3-\frac{2}{x}\right)}{x\left(2+\frac{1}{x}\right)}$

$$= \lim_{x\to\infty} \frac{3-\frac{2}{x}}{2+\frac{1}{x}}$$

$$= \frac{\lim_{x\to\infty}\left(3-\frac{2}{x}\right)}{\lim_{x\to\infty}\left(2+\frac{1}{x}\right)}$$

$$= \frac{3-\lim_{x\to\infty}\frac{2}{x}}{2+\lim_{x\to\infty}\frac{1}{x}}$$

$$= \frac{3-0}{2+0}$$

$$= \frac{3}{2} //$$

23 $\lim_{x\to\infty} \frac{\sqrt{1+4x^6}}{2-x^3} = \lim_{x\to\infty} \frac{\sqrt{x^6\left(\frac{1}{x^6}+4\right)}}{x^3\left(\frac{2}{x^3}-1\right)}$

$$= \lim_{x \to \infty} \frac{x^3 \sqrt{\frac{1}{x^6} + 4}}{x^3 \left(\frac{2}{x^3} - 1\right)}$$

$$= \lim_{x \to \infty} \frac{\sqrt{\frac{1}{x^6} + 4}}{\frac{2}{x^3} - 1}$$

$$= \frac{\sqrt{\lim_{x \to \infty} \frac{1}{x^6} + 4}}{\lim_{x \to \infty} \frac{2}{x^3} - 1}$$

$$= \frac{\sqrt{0 + 4}}{0 - 1}$$

$$= \frac{2}{-1}$$

$$= \underline{\underline{-2}}$$

(24) $$\lim_{x \to -\infty} \frac{\sqrt{1 + 4x^6}}{2 - x^3} = \lim_{x \to -\infty} \frac{\sqrt{x^6 \left(\frac{1}{x^6} + 4\right)}}{x^3 \left(\frac{2}{x^3} - 1\right)}$$

(24)

$$\lim_{x \to -\infty} \frac{\sqrt{1+4x^6}}{2-x^3} = \lim_{x \to -\infty} \frac{\sqrt{x^6\left(\frac{1}{x^6}+4\right)}}{x^3\left(\frac{2}{x^3}-1\right)}$$

$$= \lim_{x \to -\infty} \frac{-x^3\sqrt{\frac{1}{x^6}+4}}{x^3\left(\frac{2}{x^3}-1\right)}$$

$$= \lim_{x \to -\infty} \frac{-\sqrt{\frac{1}{x^6}+4}}{\frac{2}{x^3}-1}$$

$$= \frac{-\sqrt{\lim\limits_{x \to -\infty} \frac{1}{x^6}+4}}{\lim\limits_{x \to -\infty} \frac{2}{x^3}-1}$$

$$= \frac{-\sqrt{0+4}}{0-1}$$

$$= -\frac{2}{-1}$$

$$= 2$$

27 $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{9x^2+x}-3x\right)\left(\sqrt{9x^2+x}+3x\right)}{\sqrt{9x^2+x}+3x}$$

$$=\lim_{x\to\infty}\frac{(9x^2+x)-(3x)^2}{\sqrt{9x^2+x}+3x}$$

$$=\lim_{x\to\infty}\frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}$$

$$=\lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}$$

$$=\lim_{x\to\infty}\frac{x}{x\sqrt{9+\frac{1}{x}}+3x}$$

$$=\lim_{x\to\infty}\frac{x}{x\left(\sqrt{9+\frac{1}{x}}+3\right)}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{9+\frac{1}{x}}+3}$$

$$= \frac{1}{\sqrt{9+\lim_{x\to\infty}\frac{1}{x}}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3}$$

$$= \frac{1}{6}$$

31 $\lim_{x\to\infty} \frac{x^4-3x^2+x}{x^3-x+2} = \lim_{x\to\infty} \frac{x^4\left(1-\frac{3}{x^2}+\frac{1}{x^3}\right)}{x^4\left(\frac{1}{x}-\frac{1}{x^3}+\frac{2}{x^4}\right)}$

$$= \lim_{x\to\infty} \frac{1-\frac{3}{x^2}+\frac{1}{x^3}}{\frac{1}{x}-\frac{1}{x^3}+\frac{2}{x^4}}$$

$$= \lim \left(1-\frac{3}{x^2}+\frac{1}{x^3}\right)$$

$$= \frac{\lim_{x\to\infty}\left(1 - \frac{3}{x^2} + \frac{1}{x^3}\right)}{\lim_{x\to\infty}\left(\frac{1}{x} - \frac{1}{x^3} + \frac{2}{x^4}\right)}$$

$$= \frac{1 - 0 + 0}{0 - 0 + 0}$$

$$= \frac{1}{0}$$

$$= \infty$$

(Notice that $\frac{1}{x}$ is still "jogging" to 0 when $\frac{1}{x^3}$ has reached 0 so the denominator moves positively to 0.)

41 $\lim_{x\to\infty}\left[\ln(1+x^2) - \ln(1+x)\right]$

$$= \lim_{x\to\infty} \ln\left(\frac{1+x^2}{1+x}\right)$$

$$= \lim_{x\to\infty} \ln \frac{x^2\left(\frac{1}{x^2}+1\right)}{x^2\left(\frac{1}{x^2}+\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \ln \frac{\frac{1}{x^2} + 1}{\frac{1}{x^2} + \frac{1}{x}}$$

$$= \ln \frac{\lim_{x\to\infty}\left(\frac{1}{x^2} + 1\right)}{\lim_{x\to\infty}\left(\frac{1}{x^2} + \frac{1}{x}\right)}$$

$$= \ln \frac{0+1}{0+0}$$

$$= \ln \frac{1}{0}$$

$$= \ln \infty$$

$$= \infty$$

49 $y = \dfrac{2x^2 + x - 1}{x^2 + x - 2} = \dfrac{x^2\left(2 + \frac{1}{x} - \frac{1}{x^2}\right)}{x^2\left(1 + \frac{1}{x} - \frac{2}{x^2}\right)}$

$$= \frac{2 + \frac{1}{x} - \frac{1}{x^2}}{1 + \frac{1}{x} - \frac{2}{x^2}}$$

$$\longrightarrow \frac{2+0-0}{} = 2 \text{ as } x \to \pm\infty$$

$$\longrightarrow \frac{2+0-0}{1+0-0} = 2 \text{ as } x \to \pm\infty$$

So $y=2$ is a horizontal asymptote.

Also,

$$x^2+x-2=0 \Rightarrow x^2+2x-x-2=0$$

$$\Rightarrow x(x+2)-1(x+2)=0$$

$$\Rightarrow (x+2)(x-1)=0$$

$$\Rightarrow x=-2,\ x=1$$

So $x=-2$ and $x=1$ are vertical asymptotes.

50 $y = \dfrac{1+x^4}{x^2-x^4} = \dfrac{x^4\left(\frac{1}{x^4}+1\right)}{x^4\left(\frac{1}{x^2}-1\right)}$

$$= \frac{\frac{1}{x^4}+1}{\frac{1}{x^2}-1} \longrightarrow \frac{0+1}{0-1} = -1$$

as $x \to \pm\infty$

So $y=-1$ is a horizontal asymptote.