7) $y=\sqrt{x}$, $(1,1)$. Here $a=1$ and $f(a)=1$. So

$$m=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$$

$$=\lim_{x\to 1}\frac{\sqrt{x}-\sqrt{1}}{x-1}$$

$$=\lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)}$$

$$=\lim_{x\to 1}\frac{(\sqrt{x})^2-1^2}{(x-1)(\sqrt{x}+1)}$$

$$=\lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{x}+1)}$$

$$=\lim_{x\to 1}\frac{1}{\sqrt{x}+1}$$

$$=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$$

Thus, tangent line is

$$y-f(a)=m(x-a)\Rightarrow y-1=\tfrac{1}{2}(x-1)$$

$$\Rightarrow y=\tfrac{1}{2}x-\tfrac{1}{2}+1$$

$$\Rightarrow \underline{\underline{y=\tfrac{1}{2}x+\tfrac{1}{2}}}$$

⑧ $y = \dfrac{2x+1}{x+2}$, $(1,1)$. Here $a = 1$ and $f(a) = 1$. So

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{\dfrac{2x+1}{x+2} - \dfrac{2(1)+1}{1+2}}{x-1}$$

$$= \lim_{x \to 1} \frac{\dfrac{2x+1}{x+2} - 1}{x-1}$$

$$= \lim_{x \to 1} \frac{\dfrac{2x+1-1(x+2)}{x+2}}{x-1}$$

$$= \lim_{x \to 1} \left(\frac{2x+1-x-2}{x+2} \div \frac{x-1}{1} \right)$$

$$= \lim_{x \to 1} \left(\frac{x-1}{x+2} \times \frac{1}{x-1} \right)$$

$$= \lim_{x \to 1} \frac{1}{x+2}$$

$$= \frac{1}{1+2} = \frac{1}{3}$$

Thus, tangent line is

$$y - f(a) = m(x-a) \Rightarrow y - 1 = \tfrac{1}{3}(x-1)$$

$$\Rightarrow y = \tfrac{1}{3}x - \tfrac{1}{3} + 1$$

$$\Rightarrow y = \frac{1}{3}x - \frac{1}{3} + 1$$

$$\Rightarrow y = \frac{1}{3}x + \frac{2}{3}$$

⑬ $s(t) = 40t - 16t^2$. $v(2) = ?$

$$v(2) = \lim_{h\to 0} \frac{s(2+h) - s(2)}{h}$$

$$= \lim_{h\to 0} \frac{40(2+h) - 16(2+h)^2 - (40(2) - 16(2^2))}{h}$$

$$= \lim_{h\to 0} \frac{80 + 40h - 16(4 + 4h + h^2) - (80 - 64)}{h}$$

$$= \lim_{h\to 0} \frac{80 + 40h - 64 - 64h - 16h^2 - 80 + 64}{h}$$

$$= \lim_{h\to 0} \frac{40h - 64h - 16h^2}{h}$$

$$= \lim_{h\to 0} \frac{-24h - 16h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(-24 - 16h)}{h}$$

$$= \lim_{h\to 0} (-24 - 16h)$$

$$= -24 - 16(0)$$

$$= -24$$

Hence, the (instantaneous) velocity when $t = 2$ seconds is -24 ft/s.

31 $f(x) = 3x^2 - 4x + 1$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{3(a+h)^2 - 4(a+h) + 1 - (3a^2 - 4a + 1)}{h}$$

$$= \lim_{h \to 0} \frac{3(a^2 + 2ah + h^2) - 4a - 4h + 1 - 3a^2 + 4a - 1}{h}$$

$$= \lim_{h \to 0} \frac{3a^2 + 6ah + 3h^2 - 4a - 4h + 1 - 3a^2 + 4a - 1}{h}$$

$$= \lim_{h \to 0} \frac{6ah + 3h^2 - 4h}{h}$$

$$= \lim_{h \to 0} \frac{h(6a + 3h - 4)}{h}$$

$$= \lim_{h \to 0} (6a + 3h - 4)$$

$$= 6a + 3(0) - 4$$

$$= 6a - 4$$

33. $f(x) = \dfrac{2t+1}{t+3}$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{2(a+h)+1}{(a+h)+3} - \dfrac{2a+1}{a+3}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{2a+2h+1}{a+h+3} - \dfrac{2a+1}{a+3}}{h}$$

$$= \lim_{h \to 0} \frac{(a+3)(2a+2h+1) - (2a+1)(a+h+3)}{(a+h+3)(a+3)} \div \frac{h}{1}$$

$$= \lim_{h \to 0} \frac{(2a^2+2ah+a+6a+6h+3) - (2a^2+2ah+6a+a+h+3)}{(a+h+3)(a+3)} \times \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{(2a^2+2ah+7a+6h+3) - (2a^2+2ah+7a+h+3)}{h(a+h+3)(a+3)}$$

$$= \lim_{h \to 0} \frac{6h - h}{h(a+h+3)(a+3)}$$

$$= \lim_{h \to 0} \frac{5h}{h(a+h+3)(a+3)}$$

$$= \lim_{h \to 0} \frac{5}{(a+h+3)(a+3)}$$

5

$$= \frac{5}{\lim_{h \to 0} (a+h+3)(a+3)}$$

$$= \frac{5}{(a+3) \lim_{h \to 0} (a+h+3)}$$

$$= \frac{5}{(a+3)(a+0+3)}$$

$$= \frac{5}{(a+3)(a+3)} = \underline{\underline{\frac{5}{(a+3)^2}}}$$

36 $f(x) = \frac{4}{\sqrt{1-x}}$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{\sqrt{1-(a+h)}} - \frac{4}{\sqrt{1-a}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{\sqrt{1-a-h}} - \frac{4}{\sqrt{1-a}}}{h}$$

$$= \lim_{h \to 0} \frac{4\sqrt{1-a} - 4\sqrt{1-a-h}}{\sqrt{1-a-h}\,\sqrt{1-a}} \div \frac{h}{1}$$

$$= \lim \quad 4\left(\sqrt{1-a} - \sqrt{1-a-h}\right) \quad \times \quad \frac{1}{}$$

$$= \lim_{h \to 0} \frac{4\left(\sqrt{1-a} - \sqrt{1-a-h}\right)}{\sqrt{1-a-h}\ \sqrt{1-a}} \times \frac{1}{h}$$

$$= 4 \lim_{h \to 0} \frac{\sqrt{1-a} - \sqrt{1-a-h}}{h\sqrt{1-a-h}\ \sqrt{1-a}}$$

$$= 4 \lim_{h \to 0} \frac{\left(\sqrt{1-a} - \sqrt{1-a-h}\right)\left(\sqrt{1-a} + \sqrt{1-a-h}\right)}{\left(h\sqrt{1-a-h}\ \sqrt{1-a}\right)\left(\sqrt{1-a} + \sqrt{1-a-h}\right)}$$

$$= 4 \lim_{h \to 0} \frac{\left(\sqrt{1-a}\right)^2 - \left(\sqrt{1-a-h}\right)^2}{h\left(\sqrt{1-a-h}\right)\left(\sqrt{1-a}\right)\left(\sqrt{1-a} + \sqrt{1-a-h}\right)}$$

$$= 4 \lim_{h \to 0} \frac{1-a - (1-a-h)}{h\left(\sqrt{1-a-h}\right)\left(\sqrt{1-a}\right)\left(\sqrt{1-a} + \sqrt{1-a-h}\right)}$$

$$= 4 \lim_{h \to 0} \frac{1-a-1+a+h}{h\left(\sqrt{1-a-h}\right)\left(\sqrt{1-a}\right)\left(\sqrt{1-a} + \sqrt{1-a-h}\right)}$$

$$= 4 \lim_{h \to 0} \frac{h}{h\left(\sqrt{1-a-h}\right)\left(\sqrt{1-a}\right)\left(\sqrt{1-a} + \sqrt{1-a-h}\right)}$$

$$= 4 \lim_{h \to 0} \frac{1}{\left(\sqrt{1-a-h}\right)\left(\sqrt{1-a}\right)\left(\sqrt{1-a} + \sqrt{1-a-h}\right)}$$

$$= \frac{4}{\lim_{h\to 0}\left(\left(\sqrt{1-a-h}\right)\left(\sqrt{1-a}\right)\left(\sqrt{1-a}+\sqrt{1-a-h}\right)\right)}$$

$$= \frac{4}{\left(\sqrt{1-a-0}\right)\left(\sqrt{1-a}\right)\left(\sqrt{1-a}+\sqrt{1-a-0}\right)}$$

$$= \frac{4}{\left(\sqrt{1-a}\right)\left(\sqrt{1-a}\right)\left(\sqrt{1-a}+\sqrt{1-a}\right)}$$

$$= \frac{4}{\left(\sqrt{1-a}\right)\left(\sqrt{1-a}\right)\left(2\sqrt{1-a}\right)}$$

$$= \frac{2}{\left(\sqrt{1-a}\right)^{3}}$$

$$= \frac{2}{(1-a)^{3/2}}$$

37 $\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} = f'(9)$; so $f(x)=\sqrt{x}$ and $a=9$.

41 $\lim_{h\to 0} \frac{\cos(\pi+h)+1}{h} = \lim_{h\to 0} \frac{\cos(\pi+h)-(-1)}{h}$

$$= \lim_{h\to 0} \frac{\cos(\pi+h)-\cos\pi}{h}$$

$$= f'(\pi).$$

So $f(x) = \cos x$ and $a = \pi$.

Notice (also) that

$$\lim_{h \to 0} \frac{\cos(\pi + 0 + h) - \cos(\pi + 0)}{h} = f'(0) \text{ if } f(x) = \cos(\pi + x) \text{ and } a = 0.$$