7. $f(t) = 2t^3 - 3t^2 - 4t$

$$f'(t) = 2(3t^2) - 3(2t) - 4(1)$$
$$= 6t^2 - 6t - 4$$

9. $g(x) = x^2(1-2x)$

$$= x^2 - 2x^3$$
$$g'(x) = 2x - 2(3x^2)$$
$$= 2x - 6x^2$$

18. $y = \sqrt[3]{x}\,(2+x)$

$$= x^{1/3}(2+x)$$
$$= 2x^{1/3} + x^{1+\frac{1}{3}}$$
$$= 2x^{1/3} + x^{4/3}$$
$$y' = 2\left(\frac{1}{3}x^{\frac{1}{3}-1}\right) + \frac{4}{3}x^{\frac{4}{3}-1}$$
$$= \frac{2}{3}x^{-\frac{2}{3}} + \frac{4}{3}x^{\frac{1}{3}}$$
$$= \frac{2}{3x^{2/3}} + \frac{4}{3}x^{1/3}$$
$$= \frac{2}{3\sqrt[3]{x^2}} + \frac{4}{3}\sqrt[3]{x}$$

(22) $y = \dfrac{\sqrt{x} + x}{x^2}$

$$= \frac{x^{1/2}}{x^2} + \frac{x}{x^2}$$

$$= x^{\frac{1}{2}-2} + x^{1-2}$$

$$= x^{-\frac{3}{2}} + x^{-1}$$

$$y' = -\frac{3}{2}x^{-\frac{3}{2}-1} - 1x^{-1-1}$$

$$= -\frac{3}{2}x^{-\frac{5}{2}} - x^{-2}$$

$$= \frac{-3}{2x^{5/2}} - \frac{1}{x^2}$$

$$= -\frac{3}{2\sqrt{x^5}} - \frac{1}{x^2}$$

(30) $D(t) = \dfrac{1+16t^2}{(4t)^3}$

$$= \frac{1+16t^2}{64t^3}$$

$$= \frac{1}{64t^3} + \frac{16t^2}{64t^3}$$

$$= \frac{1}{64} t^{-3} + \frac{1}{4} t^{-1}$$

So

$$D'(t) = \frac{1}{64}\left(-3t^{-3-1} + \frac{1}{4}\left(-t^{-1-1}\right)\right.$$

$$= \frac{-3}{64} t^{-4} - \frac{1}{4} t^{-2}$$

$$= -\frac{3}{64 t^4} - \frac{1}{4t^2}$$

(31) $z = \frac{A}{y^{10}} + Be^y$

$$= Ay^{-10} + Be^y$$

$$z' = A\left(-10y^{-10-1} + Be^y\right.$$

$$= -10Ay^{-11} + Be^y$$

$$= -\frac{10A}{y^{11}} + Be^y$$

(32) $y = e^{x+1} + 1$

$$= e^1 \cdot e^x + 1$$

$$= e \cdot e^x + 1$$

$$y' = e \cdot e^x + 0$$

$$= e \cdot e^x$$
$$= e^{x+1}$$

35 $y(x) = x + \frac{2}{x}$, $(2,3)$

Equation of tangent line to y at $(2,3)$ is
$$y - 3 = m(x-2)$$
$$y = mx - 2m + 3$$

where
$$m = y'(2)$$

But
$$y(x) = x + \frac{2}{x} = x + 2x^{-1}$$

So
$$y'(x) = 1 + 2(-1x^{-1-1})$$
$$= 1 - 2x^{-2}$$
$$= 1 - \frac{2}{x^2}$$

$$\Rightarrow y'(2) = 1 - \frac{2}{2^2}$$
$$= 1 - \frac{1}{2}$$
$$= \frac{1}{2} = m$$

Hence,

$$y = mx - m + 3$$

$$= \frac{1}{2}x - \frac{1}{2} + 3$$

$$= \frac{1}{2}x + \frac{5}{2}$$ is the required equation.

49 $s(t) = t^3 - 3t$

a $v(t) = s'(t) = 3t^2 - 3$

$a(t) = v'(t) = 3(2t) = 6t$

b At $t = 2$ seconds, $a(2) = 6(2) = 12 m/s^2$

c $v(t) = 0 \Rightarrow 3t^2 - 3 = 0 \Rightarrow 3(t^2 - 1) = 0$

$\Rightarrow 3(t-1)(t+1) = 0$

$\Rightarrow t = 1$ since $t \geq 0$

so $a(1) = 6(1)$

$= 6 m/s^2$

55 $y = 2x^3 + 3x^2 - 12x + 1$

Tangent is horizontal $\Rightarrow y'(x) = 0$

ie;

$y'(x) = 2(3x^2) + 3(2x) - 12 = 0$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$$\Rightarrow \quad x^2 + x - 2 = 0$$

$$\Rightarrow \quad x^2 - x + 2x - 2 = 0$$

$$\Rightarrow \quad x(x-1) + 2(x-1) = 0$$

$$\Rightarrow \quad (x+2)(x-1) = 0$$

$$\Rightarrow \quad x = 1, -2$$

Furthermore,

$$y(1) = 2(1^3) + 3(1^2) - 12(1) + 1 = 2 + 3 - 12 + 1 = -6$$

$$y(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) + 1 = -16 + 12 + 24 + 1 = 21$$

Hence, the points on the curve are $(1, -6)$ and $(-2, 21)$.

56 $f(x) = e^x - 2x$

Graph of f has a horizontal tangent when $f'(x) = 0$.

ie;

$$f'(x) = e^x - 2 = 0$$

$$\Rightarrow \quad e^x = 2$$

$$\Rightarrow \quad \ln e^x = \ln 2$$

$$\Rightarrow \quad x \ln e = \ln 2$$

$$\Rightarrow \quad x = \ln 2$$

Hence, the graph of $f(x) = e^x - 2x$ has a horizontal tangent at $x = \ln 2$.

(70) $y = ax^2 + bx + c$ passes through $(2, 15)$

$\Rightarrow 15 = a(2^2) + b(2) + c$

$\Rightarrow 15 = 4a + 2b + c$

$\Rightarrow 4a + 2b + c = 15$ ——— (1)

Since

$y'(x) = 2ax + b$,

$m = 4$ at $x = 1$

$\Rightarrow 4 = y'(1) = 2a(1) + b$

$\Rightarrow 2a + b = 4$ ——— (2)

$m = -8$ at $x = -1$

$\Rightarrow -8 = y'(-1) = 2a(-1) + b$

$\Rightarrow -2a + b = -8$ ——— (3)

(2) + (3) $\Rightarrow 2b = -4$

$\Rightarrow b = -2$

Substitute $b = -2$ into (2):

$\Rightarrow 2a - 2 = 4$

$\Rightarrow 2a = 4 + 2 = 6$

$\Rightarrow \quad 2a = 6$

$\Rightarrow \quad a = 3$

Substitute $a = 3$ and $b = -2$ into ①:

$4a + 2b + c = 15$

$\Rightarrow \quad 4(3) + 2(-2) + c = 15$

$12 - 4 + c = 15$

$8 + c = 15$

$c = 15 - 8$

$= 7$

Hence, the required parabola is

$y = 3x^2 - x + 7$