3.1 Derivatives of Polynomials and Exponential Functions

In this section we learn how to differentiate constant functions, power functions, polynomials and exponential functions.

Suppose c is a constant and f and g are both differentiable functions, and n is any real number, then

1. **The Power Rule:**
$$\frac{d}{dx}(x^n) = nx^{n-1}$$
2. **Derivative of a Constant Function:**
$$\frac{d}{dx}(c) = 0$$
3. **The Constant Multiple Rule:**
$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$
4. **The Sum and Difference Rule:**
$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$
5. **Derivative of the Natural Exponential Function:** The number e is an irrational number and its approximate value is 2.71828,
$$\frac{d}{dx}(e^x) = e^x$$

Example 1 Find the derivative of the function f by using the rules of differentiation.

(a) $f(x) = 5$

(b) $f(x) = -2$

(c) $f(x) = e$

(d) $f(x) = x^3$

(e) $f(x) = x^{-12}$

(f) $f(R) = 4\pi R^2$

(g) $y = \frac{s-\sqrt{s}}{s^2}$

(h) $f(t) = \frac{2}{t^5} - \frac{3}{t^4} + \frac{7}{t} - 6$

Example 2 Find equations of the tangent line and normal line to the curve $y = x^4 + 2e^x$ at the point (0,2).

Example 3 The equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds.

(a) Find the velocity and acceleration as functions of t.

(b) Find the acceleration after 5s.

Example 4 Find the points on the curve $y = 2x^3+3x^2-12x+6$ where the tangent line is horizontal.