4 $g(x) = (x + 2\sqrt{x})e^x$

$$= (x + 2x^{1/2})e^x$$

$$g'(x) = (x + 2x^{1/2})e^x + \left(1 + 2\left(\tfrac{1}{2}x^{\frac{1}{2}-1}\right)\right)e^x$$

$$= (x + 2x^{1/2})e^x + (1 + x^{-1/2})e^x$$

$$= (x + 2x^{1/2})e^x + \left(1 + \frac{1}{x^{1/2}}\right)e^x$$

$$= \left(x + 2x^{1/2} + 1 + \frac{1}{x^{1/2}}\right)e^x$$

$$= \left(\frac{x^{3/2} + 2x + x^{1/2} + 1}{x^{1/2}}\right)e^x$$

$$= \left(\frac{\sqrt{x^3} + 2x + \sqrt{x} + 1}{\sqrt{x}}\right)e^x$$

8 $G(x) = \dfrac{x^2 - 2}{2x+1} = \dfrac{u(x)}{v(x)}$

$$\left.\begin{aligned} u'(x) &= 2x \\ v'(x) &= 2 \end{aligned}\right.$$

$$G'(x) = \frac{v(x)u'(x) - v'(x)u(x)}{(v(x))^2}$$

$$= \frac{(2x+1)2x - 2(x^2 - 2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2 + 4}{(2x+1)^2}$$

$$= \frac{2x^2 + 2x + 4}{(2x+1)^2}$$

9) $H(u) = (u - \sqrt{u})(u + \sqrt{u}) = x(u)\, y(u)$

$x(u) = u - \sqrt{u} = u - u^{1/2}$

$\Rightarrow x'(u) = 1 - \frac{1}{2} u^{-1/2}$

$y \quad x) \quad u \quad \sqrt{u} \quad - u \quad u$

$= y \quad u \quad - \quad u$

$H'(u) = x(u)\, y'(u) + x'(u)\, y(u)$

$= (u - \sqrt{u})(1 + \frac{1}{2} u^{-1/2}) + (1 - \frac{1}{2} u^{-1/2})(u + \sqrt{u})$

$= u + \frac{1}{2} u^{1/2} - \sqrt{u} - \frac{1}{2} + u + \sqrt{u} - \frac{1}{2} u^{1/2} - \frac{1}{2}$

$= u + u - \frac{1}{2} - \frac{1}{2}$

$= 2u - 1$

$/ \quad L \quad n \quad - \quad y$

$H(u) = (u - \sqrt{u})(u + \sqrt{u}) = u^2 - (\sqrt{u})^2 = u^2 - u$

$\Rightarrow H'(u) = 2u - 1$

11) $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3)$

$= (y^{-2} - 3y^{-4})(y + 5y^3)$

$= u(y)\, v(y)$

$u'(y) = -2y^{-3} - 3(-4y^{-5})$

$= -2y^{-3} + 12y^{-5}$

$v \quad - \quad 5 \quad 3y^2$

$- \quad y$

$F'(y) = u(y)\, v'(y) + u'(y)\, v(y)$

$= (y^{-2} - 3y^{-4})(1 + 15y^2) + (-2y^{-3} + 12y^{-5})(y + 5y^3)$

$$= (y^{-2} - 3y^{-4})(1 + 15y^2) + (-2y^{-3} + 12y^{-5})(y + 5y^3)$$

$$= y^{-2} + 15 - 3y^{-4} - 45y^{-2} - 2y^{-2} - 10 + 12y^{-4} + 60y^{-2}$$

$$= 14y^{-2} + 9y^{-4} + 5$$

$$= \frac{14}{y^2} + \frac{9}{y^4} + 5$$

(15) $y = \dfrac{t^3 + 3t}{t^2 - 4t + 3} = \dfrac{u(t)}{v(t)}$

$$\begin{cases} u(t) = t^3 + 3t \Rightarrow u'(t) = 3t^2 + 3 \\ v(t) = t^2 - 4t + 3 \\ \quad \Rightarrow v'(t) = 2t - 4 \end{cases}$$

$$y'(t) = \frac{v(t)u'(t) - v'(t)u(t)}{(v(t))^2}$$

$$= \frac{(t^2 - 4t + 3)(3t^2 + 3) - (2t - 4)(t^3 + 3t)}{(t^2 - 4t + 3)^2}$$

$$= \frac{3t^4 + 3t^2 - 12t^3 - 12t + 9t^2 + 9 - (2t^4 + 6t^2 - 4t^3 - 12t)}{(t^2 - 4t + 3)^2}$$

$$= \frac{t^4 + 6t^2 - 8t^3 + 9}{(t^2 - 4t + 3)^2}$$

$$= \frac{t^4 - 8t^3 + 6t^2 + 9}{(t^2 - 4t + 3)^2}$$

$$=$$

(17) $y = e^p(p + p\sqrt{p}) = u(p)\,v(p)$

$u(p) = e^p \Rightarrow u'(p) = e^p$

$v(p) = p + p \cdot p^{1/2}$

$= p + p^{3/2}$

$\Rightarrow v'(p) = 1 + \frac{3}{2}p^{1/2}$

$= 1 + \frac{3}{2}\sqrt{p}$

$y' = u(p)v'(p) + u'(p)v(p)$

$= e^p(1 + \frac{3}{2}\sqrt{p}) + e^p(p + p\sqrt{p})$

$= e^p(1 + \frac{3}{2}\sqrt{p} + p + p\sqrt{p})$

(18) $h(r) = \dfrac{ae^r}{b+e^r} = \dfrac{u(r)}{v(r)}$

$u(r) = ae^r \Rightarrow u'(r) = ae^r$

$v(r) = b + e^r \Rightarrow v'(r) = e^r$

$$h'(r) = \frac{v(r)u'(r) - v'(r)u(r)}{(v(r))^2}$$

$$= \frac{(b+e^r)ae^r - e^r(ae^r)}{(b+e^r)^2}$$

$$= \frac{abe^r + ae^{2r} - ae^{2r}}{(b+e^r)^2}$$

$$= \frac{abe^r}{(b+e^r)^2}$$

(20) $y = (z^2+e^z)\sqrt{z}$

$= (z^2+e^z)z^{1/2}$

$= u(z)v(z)$

$u(z) = z^2+e^z \Rightarrow u'(z) = 2z+e^z$

$v(z) = z^{1/2} \Rightarrow v'(z) = \frac{1}{2}z^{-1/2}$

$$y'(z) = u(z)v'(z) + u'(z)v(z)$$

$$= (z^2+e^z)\frac{1}{2}z^{-1/2} + (2z+e^z)z^{1/2}$$

$$= \frac{z^2+e^z+(2z+e^z)2z}{2z^{1/2}}$$

$$= \frac{5z^2+e^z+2ze^z}{2\sqrt{z}}$$

(33) $y = 2xe^x$, $(0,0)$

$$y' = 2x(e^x) + (2)e^x$$

$$= 2xe^x + 2e^x$$

$$= (2x+1)e^x$$

$$m_t = y'(0) = (2(0)+1)e^0$$

$$= 1$$

$$m_n = \frac{-1}{m_t} = -\frac{1}{1} = -1$$

Equation of tangent line:

$y - 0 = m_t(x-0)$

$\Rightarrow y = x$

Equation of normal line:

$y - 0 = m_n(x-0)$

$\Rightarrow y = -x$

(34) $y = \dfrac{2x}{x^2+1}$, $(1,1)$

Equation of tangent line:

(34) $y = \frac{2x}{x^2+1}$, $(1,1)$

$$y'(x) = \frac{(x^2+1)2 - 2x(2x)}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

$$= \frac{-2x^2+2}{(x^2+1)^2}$$

$$m_t = y'(1) = \frac{-2(1^2)+2}{(1^2+1)^2}$$

$$= 0$$

$m_n = -\frac{1}{0}$ = undefined.

Equation of tangent line:

$y-1 = m_t(x-1)$

$\Rightarrow y = 0(x-1)+1$

$\Rightarrow y = 1$

We note that m_n = undefined

$\Rightarrow$ normal line to the curve at $(1,1)$ is a vertical line

$\Rightarrow$ $x = 1$ is the normal line.

(44) $f(4)=2$, $g(4)=5$, $f'(4)=6$, $g'(4)=-3$. Find $h'(4)$

a $h(x) = 3f(x) + 8g(x)$

$h'(x) = 3f'(x) + 8g'(x)$

$\Rightarrow h'(4) = 3f'(4) + 8g'(4)$

$= 3(6) + 8(-3)$

$= 18 - 24$

$= -6$

b $h(x) = f(x)g(x)$

$h'(x) = f(x)g'(x) + f'(x)g(x)$

$\Rightarrow h'(x) = f(4)g'(4) + f'(4)g(4)$

$= 2(-3) + 6(5)$

$= -6 + 30$

$= 24$

$h(x) = \frac{f(x)}{g(x)}$

$h(x) = \frac{g(x)}{f(x)+g(x)}$

$$h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$$

$$\Rightarrow h'(x) = \frac{g(4)f'(4) - g'(4)f(4)}{(g(x))^2}$$

$$= \frac{5(6) - (-3)(2)}{5^2}$$

$$= \frac{30 + 6}{25}$$

$$= \frac{36}{25}$$

$$h(x) = \frac{g(x)}{f(x) + g(x)}$$

$$h'(x) = \frac{(f(x)+g(x))g'(x) - g(x)(f'(x)+g'(x))}{(f(x)+g(x))^2}$$

$$\Rightarrow h'(4) = \frac{(f(4)+g(4))g'(4) - g(4)(f'(4)+g'(4))}{(f(4)+g(4))^2}$$

$$= \frac{(2+5)(-3) - 5(6-3)}{(2+5)^2}$$

$$= \frac{7(-3) - 5(3)}{7^2}$$

$$= \frac{-21 - 15}{49} = \frac{-36}{49} //$$

45 $f(x) = e^x g(x)$, $g(0) = 2$, $g'(0) = 5$, $f'(0) = ?$

$$f'(x) = e^x g'(x) + e^x g(x)$$

$$\Rightarrow f'(0) = e^0 g'(0) + e^0 g(0)$$

$$= 1(5) + 1(2)$$

$$= 5 + 2$$

$$= 7$$