3.2 The Product and Quotient Rules

In this section we learn how to differentiate using the **product** and **quotient** rules.

Let f and g be both differentiable functions. Then

1. **The Product Rule:**

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

2. **The Quotient Rule:**

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\,\frac{d}{dx}[f(x)] - f(x)\,\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Example 1 Find the derivative of the given function by using the appropriate rules of differentiation.

(a) $f(x) = (3x^2 - 5x)e^x$

(b) $y = \frac{x}{e^x}$

(c) $g(t) = \sqrt{t}\,e^t$

(d) $f(x) = \frac{x^2}{1+e^x}$

(e) $f(x) = \frac{x}{x^2-1}$

(f) $G(x) = \frac{x^2-2}{2x+1}$

(g) $g(x) = e^x(x + x\sqrt{x})$

(h) $J(v) = (v^3 - 2v)(v^{-4} + v^{-2})$

Example 2 Find equations of the tangent line to the curve $y = \frac{1+x}{1+e^x}$ at the point $(0, \frac{1}{2})$.

Example 3 If $f(x) = e^x\,g(x)$, where $g(0) = 2$ and $g'(0) = 5$, find $f'(0)$.

Example 4 Suppose that $f(4) = 2,\ g(4) = 5,\ f'(4) = 6$, and $g'(4) = -3$. Find $h'(4)$ if

$$h(x) = \frac{g(x)}{f(x) + g(x)}$$

Example 5 Find $f'(x)$ and $f''(x)$ if $f(x) = \sqrt{x}\,e^x$