Section 3.3

③ $f(x) = e^x \cos x$

$= u(x)\, v(x)$

$\begin{cases} u(x) = e^x \Rightarrow u'(x) = e^x \\ v(x) = \cos x \Rightarrow v'(x) = -\sin x \end{cases}$

$f'(x) = u(x)v'(x) + u'(x)v(x)$

$= e^x(-\sin x) + e^x \cos x$

$= e^x(-\sin x + \cos x)$

$= e^x(\cos x - \sin x)$

⑥ $g(\theta) = e^\theta(\tan\theta - \theta)$

$= u(\theta)\, v(\theta)$

$\begin{cases} u(\theta) = e^\theta \Rightarrow u'(\theta) = e^\theta \\ v(\theta) = \tan\theta - \theta \\ \Rightarrow v'(\theta) = \sec^2\theta - 1 \end{cases}$

$g'(\theta) = u'(\theta)v(\theta) + u(\theta)v(\theta)$

$= e^\theta(\tan\theta - \theta) + e^\theta(\sec^2\theta - 1)$

$= e^\theta(\tan\theta - \theta + \sec^2\theta - 1)$

$= e^\theta(\tan\theta + \sec^2\theta - \theta - 1)$

⑫ $y = \dfrac{\cos x}{1 - \sin x}$

$u(x) = \cos x \Rightarrow u'(x) = -\sin x$

$v(x) = 1 - \sin x \Rightarrow v'(x) = -\cos x$

$= \dfrac{u(x)}{v(x)}$

$$= \frac{u(x)}{v(x)}$$

$$y' = \frac{v(x)\,u'(x) - v'(x)\,u(x)}{\left(v(x)\right)^2}$$

$$= \frac{(1-\sin x)(-\sin x) - (-\cos x)(\cos x)}{(1-\sin x)^2}$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$$

$$= \frac{1-\sin x}{(1-\sin x)^2}$$

$$= \frac{1}{1-\sin x}$$

(14) $y = \dfrac{\sin t}{1+\tan t}$

$= \dfrac{u(t)}{v(t)}$

$u(t) = \sin t \Rightarrow u'(t) = \cos t$

$v(t) = 1+\tan t$

$\Rightarrow v'(t) = \sec^2 t$

$$y' = \frac{v(t)\,u'(t) - v'(t)\,u(t)}{\left(v(t)\right)^2}$$

$$= \frac{(1+\tan t)\cos t - \sec^2 t \sin t}{(\ldots)^2}$$

$$= \frac{(1+\tan t)\cos t - \sec^2 t \sin t}{(1+\tan t)^2}$$

$$= \frac{\cos t + \frac{\sin t}{\cos t}\cdot \cos t - \frac{1}{\cos^2 t}\cdot \sin t}{(1+\tan t)^2}$$

$$= \frac{\cos t + \sin t - \sec t \tan t}{(1+\tan t)^2}$$

(15) $f(\theta) = \theta \cos\theta \sin\theta$

$= u(\theta)v(\theta)$

$u(\theta) = \theta \cos\theta$

$\Rightarrow u'(\theta) = \theta(-\sin\theta) + 1(\cos\theta)$

$= -\theta\sin\theta + \cos\theta$

$v(\theta) = \sin\theta \Rightarrow v'(\theta) = \cos\theta$

$$f'(\theta) = u'(\theta)v(\theta) + u(\theta)v'(\theta)$$

$$= (\cos\theta - \theta\sin\theta)\sin\theta + \theta\cos\theta(\cos\theta)$$

$$= \cos\theta\sin\theta - \theta\sin^2\theta + \theta\cos^2\theta$$

$$= \frac{1}{2}\sin(2\theta) + \theta(\cos^2\theta - \sin^2\theta)$$

$$= \frac{1}{2}\sin(2\theta) + \theta\cos(2\theta)$$

Recall:

$\sin(2\theta) = \sin(\theta+\theta)$

$= \sin\theta\cos\theta + \sin\theta\cos\theta$

$= 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos(\theta+\theta)$

$= \cos\theta\cos\theta - \sin\theta\sin\theta$

$= \cos^2\theta - \sin^2\theta$

$= \cos\theta\cos\theta - \sin\theta\sin\theta$

$= \cos^2\theta - \sin^2\theta$

More generally:

$\sin(x+y) = \sin x \cos y + \sin y \cos x$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

(19) $\frac{d}{dx}(\cot x) = -\csc^2 x$ since $\cot x = \frac{\sin x}{\cos x}$

$$\Rightarrow \frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x(-\cos x) - (\sin x)\sin x}{(\cos x)^2}$$

$$= \frac{-\cos^2 x - \sin^2 x}{\cos^2 x}$$

$$= -\frac{(\cos^2 x + \sin^2 x)}{\cos^2 x}$$

$$= \frac{-1}{\cos^2 x}$$

$$= -\csc^2 x.$$

(22) $y = e^x \cos x$, $(0,1)$.

Equation of tangent line to curve at $(0,1)$:

Equation of tangent line to curve at $(0,1)$:

$$y - 1 = m(x - 0)$$

$$\Rightarrow y = mx + 1$$

where

$$m = y'(0).$$

But

$$y'(x) = e^{x}\cos x + e^{x}(-\sin x)$$

$$= e\,(\cos x - \sin x)$$

$$\Rightarrow \quad m = y'(0) = e^{0}(\cos 0 - \sin 0) = 1(1-0) = 1$$

Hence,

$$y = mx + 1$$

$$= x + 1$$

(40) $$\lim_{x \to 0} \frac{\sin x}{\sin \pi x} = \lim_{x \to 0} \frac{\sin x}{\sin \pi x} \cdot \frac{\pi x}{\pi x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{\pi x}{\sin \pi x} \cdot \frac{1}{\pi}$$

$$= \frac{1}{\pi} \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{\pi x}{\sin \pi x}$$

$$= \frac{1}{\pi} \cdot 1 \cdot \lim_{x \to 0} \frac{1}{\sin \pi x / \pi x}$$

$$= \frac{1}{\pi} \cdot \frac{1}{\lim\limits_{x \to 0} \frac{\sin \pi x}{\pi x}}$$

$$= \frac{1}{\pi} \cdot \frac{1}{1}$$

$$= \frac{1}{\pi} \,//$$

43 $\lim\limits_{x \to 0} \frac{\sin 3x}{5x^3 - 4x} = \lim\limits_{x \to 0} \frac{\sin 3x}{3x\left(\frac{5}{3}x^2 - \frac{4}{3}\right)}$

$$= \lim\limits_{x \to 0} \frac{\sin 3x}{3x} \cdot \frac{1}{\frac{5}{3}x^2 - \frac{4}{3}}$$

$$= \lim\limits_{x \to 0} \frac{\sin 3x}{3x} \cdot \lim\limits_{x \to 0} \frac{1}{\frac{5}{3}x^2 - \frac{4}{3}}$$

$$= 1 \cdot \frac{1}{\lim\limits_{x \to 0} \left(\frac{5}{3}x^2 - \frac{4}{3}\right)}$$

$$= 1 \cdot \frac{1}{0 - \frac{4}{3}}$$

$$= -\frac{3}{4} \,//$$

48 $$\lim_{x\to 0} \frac{\sin(x^2)}{x} = \lim_{x\to 0} \frac{\sin(x^2)}{x} \cdot \frac{x}{x}$$

$$= \lim_{x\to 0} \frac{\sin(x^2)}{x^2} \cdot \frac{x}{1}$$

$$= \lim_{x\to 0} \frac{\sin(x^2)}{x^2} \cdot \lim_{x\to 0} \frac{x}{1}$$

$$= 1 \cdot \frac{0}{1}$$

$$= 0 \;//$$

50 $$\lim_{x\to 1} \frac{\sin(x-1)}{x^2+x-2} = \lim_{x\to 1} \frac{\sin(x-1)}{x^2+2x-x-2}$$

$$= \lim_{x\to 1} \frac{\sin(x-1)}{x(x+2)-1(x+2)}$$

$$= \lim_{x\to 1} \frac{\sin(x-1)}{(x+2)(x-1)}$$

$$= \lim_{x\to 1} \frac{1}{x+2} \cdot \frac{\sin(x-1)}{x-1}$$

$$= \lim_{x\to 1} \frac{1}{x+2} \cdot \lim_{x\to 1} \frac{\sin(x-1)}{x-1}$$

$$= \frac{1}{1+2} \cdot \lim_{y\to 0} \frac{\sin y}{y} \quad \text{where } y = x-1$$

$$= \frac{1}{3} \cdot 1$$

$$= \frac{1}{3}$$ //