Lecture Note 12 (Ref. text book page 197) 09/17/2021

3.4 The Chain Rule

The Chain Rule If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and F' is given by the product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

Example Find the derivative of the following functions by using the rules of differentiation.

(a) $F(x) = \sqrt[3]{1+4x}$

(b) $g(\theta) = \cos^2 \theta$

(c) $f(\theta) = \cos(\theta)^2$

(d) $y = (x^3 - 1)^{100}$

(e) $f(x) = \dfrac{1}{\sqrt[3]{x^2 + x + 1}}$

(f) $g(t) = \left(\dfrac{t-2}{2t+1}\right)^9$

(g) $y = (2x+1)^5(x^3-x+1)^4$

(h) $y = e^{\sin x}$

(i) $f(x) = \sin(\cos(\tan x))$

(j) $y = e^{\sin 3\theta}$

3.3 Derivatives of Trigonometric Functions (Cont'd)

Later in this course, you will know (easily) why this limit holds. For now, just take it that:

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$

Example 1 Evaluate the following

(a) $\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta}$

(b) $\lim_{\theta \to 0} \frac{\cos \theta - 1}{\sin \theta}$

(c) $\lim_{t \to 0} \frac{\tan(6t)}{\sin(2t)}$

(d) $\lim_{x \to 0} \frac{\sin(3x)\,\sin(5x)}{x^2}$

(e) $\lim_{x \to 0} x\,\cot(5x)$

(f) $\lim_{x \to 1} \frac{\sin(x-1)}{x^2+x-2}$

Example 2 Find the 89th derivative of $f(x) = \cos x$