5 $x^2 - 4xy + y^2 = 4$

$$\Rightarrow \frac{d}{dx}(x^2 - 4xy + y^2) = \frac{d}{dx}(4)$$

$$\Rightarrow 2x - 4xy' - 4y + 2yy' = 0$$

$$\Rightarrow -4xy' + 2yy' = 4y - 2x$$

$$\Rightarrow y'(2y - 4x) = 4y - 2x$$

$$\Rightarrow y' = \frac{4y-2x}{2y-4x} = \frac{2y-x}{y-2x}$$

11 $y\cos x = x^2 + y^2$

$$\Rightarrow \frac{d}{dx}(y\cos x) = \frac{d}{dx}(x^2 + y^2)$$

$$\Rightarrow y'\cos x - y\sin x = 2x + 2yy'$$

$$\Rightarrow y'\cos x - 2yy' = 2x + y\sin x$$

$$\Rightarrow y'(\cos x - 2y) = 2x + y\sin x$$

$$\Rightarrow y' = \frac{2x + y\sin x}{\cos x - 2y}$$

$=$

15. $e^{x/y} = x - y$

$\Rightarrow \frac{d}{dx}\left(e^{x/y}\right) = \frac{d}{dx}(x-y)$

$\Rightarrow \frac{d}{dx}\left(e^{xy^{-1}}\right) = 1 - y'$

$\Rightarrow e^{xy^{-1}}\left(\frac{d}{dx}\left(xy^{-1}\right)\right) = 1 - y'$

$\Rightarrow e^{x/y}\left(y^{-1} - xy^{-2}y'\right) = 1 - y'$

$\Rightarrow y' - \frac{xy'}{y^2}e^{x/y} = 1 - \frac{e^{x/y}}{y}$

$\Rightarrow y^2y' - xy'e^{x/y} = y^2 - ye^{x/y}$

$\Rightarrow y'\left(y^2 - xe^{x/y}\right) = y^2 - ye^{x/y}$

$\Rightarrow y' = \frac{y^2 - ye^{x/y}}{y^2 - xe^{x/y}}$

(21) $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, $f'(1) = ?$

$$\Rightarrow \frac{d}{dx}f(x) + \frac{d}{dx}\left(x^2[f(x)]^3\right) = \frac{d}{dx}(10)$$

$$\Rightarrow f'(x) + 2x[f(x)]^3 + x^2\left(3[f(x)]^2 f'(x)\right) = 0$$

$$\Rightarrow f'(x) + 3x^2[f(x)]^2 f'(x) = -2x[f(x)]^3$$

$$\Rightarrow f'(x)\left(1 + 3x^2[f(x)]^2\right) = -2x[f(x)]^3$$

$$\Rightarrow f'(x) = \frac{-2x[f(x)]^3}{1 + 3x^2[f(x)]^2}$$

$$\Rightarrow f'(1) = \frac{-2(1)[f(1)]^3}{1 + 3(1^2)[f(1)]^2}$$

$$= \frac{-2(2^3)}{1 + 3(2^2)} = \frac{-16}{13}$$

(25) $y\sin 2x = x\cos 2y$, $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$

$$\frac{d}{dx}(y\sin 2x) = \frac{d}{dx}(x\cos 2y)$$

$$\Rightarrow \quad y'\sin 2x + 2y\cos 2x = \cos 2y - 2x\sin 2y\, y'$$

$$\Rightarrow \quad y'\sin 2x + 2x\sin 2y\, y' = \cos 2y - 2y\cos 2x$$

$$\Rightarrow \quad y'(\sin 2x + 2x\sin 2y) = \cos 2y - 2y\cos 2x$$

$$\Rightarrow \quad y' = \frac{\cos 2y - 2y\cos 2x}{\sin 2x + 2x\sin 2y}$$

$$\Rightarrow \quad m = y'(\pi/2) = \frac{\cos 2(\pi/4) - 2(\pi/4)\cos 2(\pi/2)}{\sin 2(\pi/2) + 2(\pi/2)\sin 2(\pi/4)}$$

$$= \frac{\cos\frac{\pi}{2} - \frac{\pi}{2}\cos\pi}{\sin\pi + \pi\sin\frac{\pi}{2}}$$

$$= \frac{0 + \frac{\pi}{2}}{0 + \pi}$$

$$= \frac{\pi/2}{\pi} = \frac{1}{2}$$

So equation of tangent line to curve at $(\pi/2, \pi/4)$:

$$y - \frac{\pi}{4} = m\left(x - \frac{\pi}{2}\right)$$

$$y = \frac{1}{2}\left(x - \frac{\pi}{2}\right) + \frac{\pi}{4}$$

$$= \frac{1}{2}x - \frac{\pi}{4} + \frac{\pi}{4}$$

$$= \frac{1}{2}x$$