Lecture Note 15 (Ref. text book page 208)

09/20/2021

3.5 Implicit Differentiation

The functions that we have met so far can be described by expressing one variable explicitly in terms of another variable- for example $y = \sqrt{x^3+1}$ or $y = x \sin x$, or in general, $y = f(x)$. Some functions, however, are defined implicitly by a relation between x and y such as $x^2 + y^2 = 25$ or $x^3 + y^3 = 6xy$. Fortunately, we don't need to solve an equation for y in terms of x in order to find the derivative of y. Instead we can use the method of **implicit differentiation**. This consists of differentiating both sides of the equation with respect to x and then solving the resulting equation for $y^{'}$.

Example 1.

(a) if $x^2 + y^2 = 25$, find the derivative $\frac{dy}{dx}$ by using implicit differentiation.

(b) Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

Example 2. Find the derivative y' by using implicit differentiation if

(a) $x^3 + y^3 = 6xy$,

(b) $\sin(x + y) = y^2 \cos x$

Example 3. Find the derivative y'' by using implicit differentiation if $x^4 + y^4 = 16$

Derivatives of Inverse Trigonometric Functions

The inverse trigonometric functions that occur most frequently are the ones that we have just discussed. The derivatives of the remaining four are given in the following table. The proofs of the formulas are left as exercises.

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

Example 1. (a) if $x^2 + y^2 = 25$, find the derivative $\frac{dy}{dx}$ by using implicit differentiation.

(b) Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

Example 2. Find the derivative y' by using implicit differentiation if

(a) $x^3 + y^3 = 6xy$,

(b) $\sin(x + y) = y^2 \cos x$

Example 3. Find the derivative y'' by using implicit differentiation if $x^4 + y^4 = 16$