② $f(x) = x\ln x - x$

$$\Rightarrow f'(x) = x\left(\frac{1}{x}\right) + 1\cdot\ln x - 1$$

$$= 1 + \ln x - 1$$

$$= \underline{\underline{\ln x}}$$

⑤ $f(x) = \ln\frac{1}{x}$

$$= \ln x^{-1}$$

$$= -\ln x$$

$$\Rightarrow f'(x) = -\left(\frac{1}{x}\right) = \underline{\underline{-\frac{1}{x}}}$$

⑪ $F(t) = (\ln t)^2 \sin t$

$$\Rightarrow F'(t) = (\ln t)^2 \cos t + 2\left(\frac{1}{t}\right)\ln t \sin t$$

$$= (\ln t)^2 \cos t + \frac{2\ln t \sin t}{t}$$

$$= \underline{\underline{\ln t\left(\ln t \cos t + \frac{2\sin t}{t}\right)}}$$

(13) $G(y) = \ln \frac{(2y+1)^5}{\sqrt{y^2+1}} = \ln(2y+1)^5 - \ln\sqrt{y^2+1}$

$$= 5\ln(2y+1) - \ln(y^2+1)^{1/2}$$

$$= 5\ln(2y+1) - \frac{1}{2}\ln(y^2+1)$$

$$\Rightarrow G'(y) = 5\cdot\frac{2}{2y+1} - \frac{1}{2}\cdot\frac{2y}{y^2+1}$$

$$= \frac{10}{2y+1} - \frac{y}{y^2+1}$$

$$=$$

(19) $y = \ln(e^{-x} + xe^{-x}) = \ln\left(e^{-x}(1+x)\right)$

$$= \ln e^{-x} + \ln(1+x)$$

$$= -x\ln e + \ln(1+x)$$

$$= -x + \ln(1+x)$$

$$\Rightarrow y' = -1 + \frac{1}{1+x} = \frac{-1-x+1}{1+x} = \frac{-x}{1+x}$$

(23) $y = \sqrt{x}\ln x = x^{1/2}\ln x$

$\Rightarrow y' = x^{1/2}\left(\frac{1}{x}\right) + \frac{1}{2}x^{-1/2}\ln x$

$$\Rightarrow y' = x^{1/2}\left(\frac{1}{x}\right) + \frac{1}{2}x^{-1/2}\ln x$$

$$= x^{-1/2} + \frac{1}{2}x^{-1/2}\ln x$$

$$= x^{-1/2}\left(1 + \frac{1}{2}\ln x\right)$$

$$\Rightarrow y'' = x^{-1/2}\left(0 + \frac{1}{2}\cdot\frac{1}{x}\right) - \frac{1}{2}x^{-3/2}\left(1 + \frac{1}{2}\ln x\right)$$

$$= \frac{1}{2}\cdot x^{-3/2} - \frac{1}{2}x^{-3/2}\left(1 + \frac{1}{2}\ln x\right)$$

$$= \frac{1}{2}x^{-3/2}\left(1 - 1 - \frac{1}{2}\ln x\right)$$

$$= \frac{1}{2x^{3/2}}\left(-\frac{1}{2}\ln x\right)$$

$$= -\frac{\ln x}{4\sqrt{x^3}} \quad \left(= \frac{-\ln x}{4x\sqrt{x}}\right)$$

39 $y = (x^2+2)^2(x^4+4)^4 \Rightarrow \ln y = \ln (x^2+2)^2(x^4+4)^4$

$$\Rightarrow \ln y = \ln(x^2+2)^2 + \ln(x^4+4)^4$$

$$= 2\ln(x^2+2) + 4\ln(x^4+4)$$

So

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(2\ln(x^2+2) + 4\ln(x^4+4)\right)$$

$$\Rightarrow \quad \frac{y'}{y} = 2\cdot\frac{2x}{x^2+2} + 4\cdot\frac{4x^3}{x^4+4}$$

$$\Rightarrow \quad y' = y\left(\frac{4x}{x^2+2} + \frac{16x^3}{x^4+4}\right)$$

$$= (x^2+2)^2(x^4+4)^4\left(\frac{4x}{x^2+2} + \frac{16x^3}{x^4+4}\right)$$

$$=$$

41 $y = \sqrt{\dfrac{x-1}{x^4+1}} \Rightarrow \ln y = \ln\sqrt{\dfrac{x-1}{x^4+1}}$

$$= \ln\left(\frac{x-1}{x^4+1}\right)^{1/2}$$

$$= \frac{1}{2}\ln\frac{x-1}{x^4+1}$$

$$= \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x^4+1)$$

So

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(\frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x^4+1)\right)$$

$$\qquad 4x^3$$

$$\Rightarrow \frac{y'}{y} = \frac{1}{2}\frac{1}{x-1} - \frac{1}{2}\cdot\frac{4x^3}{x^4+1}$$

$$\Rightarrow y' = y\left(\frac{1}{2(x-1)} - \frac{2x^3}{x^4+1}\right)$$

$$= \sqrt{\frac{x-1}{x^4+1}}\left(\frac{1}{2(x-1)} - \frac{2x^3}{x^4+1}\right)$$

43 $y = x^x \Rightarrow \ln y = \ln x^x = x\ln x$

so

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x\ln x)$$

$$\Rightarrow \frac{y'}{y} = x\left(\frac{1}{x}\right) + 1\cdot\ln x$$

$$\Rightarrow y' = y(1+\ln x)$$

$$= x^x(1+\ln x)$$

45 $y = x^{\sin x} \Rightarrow \ln y = \ln x^{\sin x} = \sin x \ln x$

(45) $y = x^{\sin x} \Rightarrow \ln y = \ln x^{\sin x} = \sin x \ln x$

So

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\sin x \ln x)$$

$$\Rightarrow \frac{y'}{y} = \sin x \left(\frac{1}{x}\right) + \cos x \ln x$$

$$\Rightarrow y' = y\left(\frac{\sin x}{x} + \cos x \ln x\right)$$

$$= x^{\sin x}\left(\frac{1}{x}\sin x + \cos x \ln x\right)$$

$$= x^{\sin x - 1}\left(\sin x + x\cos x \ln x\right)$$