② $f(t) = \dfrac{9t}{t^2+9}$, $s = f(t)$, $t \geq 0$.

ⓐ $s(t) = f(t) = \dfrac{9t}{t^2+9}$

$$\Rightarrow v(t) = s'(t) = \frac{(t^2+9)9 - 9t(2t)}{(t^2+9)^2}$$

$$= \frac{9t^2+81-18t^2}{(t^2+9)^2}$$

$$= \frac{81-9t^2}{(t^2+9)^2} \text{ ft/s}$$

ⓑ Velocity after 1 second:

$$v(1) = \frac{81-9(1^2)}{(1^2+9)^2} = \frac{81-9}{10^2} = \frac{72}{100} = 0.72 \text{ ft/s}$$

ⓒ Particle is at rest when $v(t) = 0$

ie;

$$\frac{81-9t^2}{(t^2+9)^2} = 0 \Rightarrow 81-9t^2 = 0$$

$$\Rightarrow (9-3t)(9+3t) = 0$$

$$\Rightarrow 9-3t = 0 \text{ or } 9+3t = 0$$

$$\Rightarrow t = 3 \text{ or } t = -3$$

Hence, the particle is at rest at $t = 3$ seconds since $t \geq 0$.

d. The particle is moving in the positive direction when $v(t) > 0$.

i.e.,
$$\frac{81 - 9t^2}{(t^2+9)^2} > 0$$

$$\Rightarrow \quad 81 - 9t^2 > 0 \quad \text{since } (t^2+9)^2 > 0$$

$$\Rightarrow \quad 9(9 - t^2) > 0$$

$$\Rightarrow \quad (3-t)(3+t) > 0$$

$$\Rightarrow \quad 0 \le t < 3 \quad \text{since } t \geq 0$$

		-3		3	
$3-t$	$+$		$+$		$-$
$3+t$	$-$		$+$		$+$
$(3-t)(3+t)$	$-$		$+$		$-$

i.e., $-3 < t < 3$

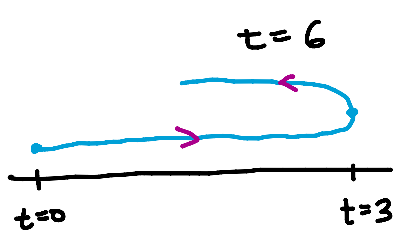
Hence, the particle is moving in the positive direction whenever whenever time, t lies in $[0, 3)$.

e. From (d) above, we see that the particle changes direction after $t = 3$ seconds, so we will compute the total distance as sum of distances over $[0, 3]$ and $[3, 6]$.

$$|s(3) - s(0)| = \frac{9(3)}{3^2+9} - \frac{9(0)}{0^2+9} = \frac{27}{18} = \frac{3}{2}$$

$$|s(6) - s(3)| = \frac{9(6)}{6^2+9} - \frac{9(3)}{3^2+9} = \frac{54}{45} - \frac{3}{2} = \frac{9}{5} - \frac{3}{2} = \frac{3}{10}$$

Hence, total distance $= \frac{3}{2} + \frac{3}{10} = \frac{18}{10} = 1.8$ feet



5 $a(t) = v'(t)$

$$= \left(\frac{81-9t^2}{(t^2+9)^2} \right)'$$

$$= \frac{(t^2+9)^2(-18t) - (4t(t^2+9))(81-9t^2)}{[(t^2+9)^2]^2}$$

$$= \frac{(t^2+9)\left((t^2+9)(-18t) - 4t(81-9t^2)\right)}{[(t^2+9)^2]^2}$$

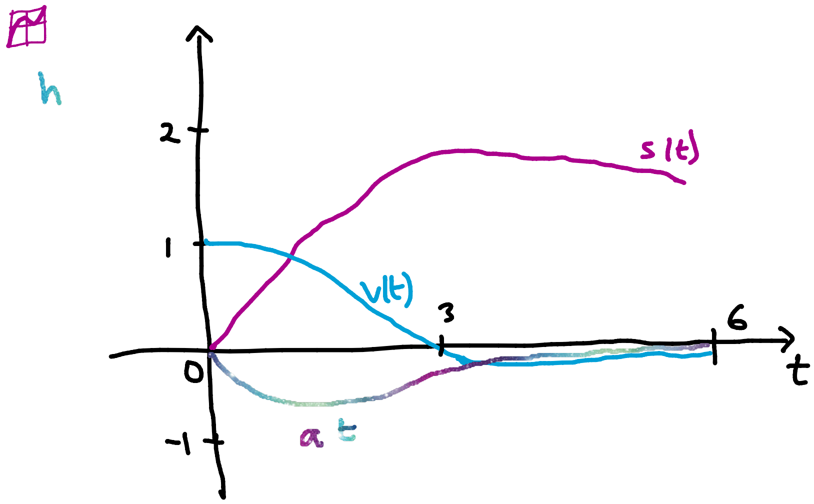
$$= \frac{-18t^3 - 162t - 324t + 36t^3}{(t^2+9)^3}$$

$$= \frac{18t^3 - 486t}{(t^2+9)^3} \text{ ft/s}^2$$

So after $t = 1$ second,

$$a(1) = \frac{18(1^3) - 486(1)}{(1^2+9)^3} = \frac{18-486}{10^3} = \frac{-468}{1000}$$

$$= -0.468 \text{ ft/s}^2$$



The particle is speeding up when v(t) and a(t) have the same sign.

$v(t) > 0$

		-3		3	
$3-t$	$+$		$+$		$-$
$3+t$	$-$		$+$		$+$
$(3-t)(3+t)$	$-$		$+$		$-$

$a(t) > 0$

		$-\sqrt{27}$		0		$\sqrt{27} \approx 5.2$	
t	$-$		$-$		$+$		$+$
$t-\sqrt{27}$	$-$		$-$		$-$		$+$
$t+\sqrt{27}$	$-$		$+$		$+$		$+$
$t(t-\sqrt{27})(t+\sqrt{27})$	$-$		$+$		$-$		$+$

Clearly, the results of these tables intersect when $3 < t < \sqrt{27}$, $t \geq 0$, so the particle is speeding at this interval.

On the other hand, the particle is slowing down when a(t) and v(t) have opposite signs and from the table, this happens when $0 < t < 3$ and $t > \sqrt{27}$.

4 Repeat similar argument when

$s(t) = t^2 e^{-t}$

(4) Repeat similar argument when

$$s = f(t) = t^2 e^{-t}$$