(4) $A = l \times w$, $\frac{dl}{dt} = 8\,cm/s$, $\frac{dw}{dt} = 3\,cm/s$, $\frac{dA}{dt} = ?$ at $l = 20\,cm$ and $w = 10\,cm$.

$$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$$

$$= 20(3) + 10(8)$$

$$= 60 + 80$$

$$= 140$$

Hence, the area of the rectangle is increasing at the rate of $140\,cm^2/s$.

(5) $V = \pi r^2 h = \pi(5^2)h = 25\pi h$

$$\Rightarrow \frac{dV}{dt} = 25\pi\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{25\pi}\frac{dV}{dt} = \frac{1}{25\pi}(3) = \frac{3}{25\pi}\,m/min$$

(14) Surface area of snowball $S = 4\pi r^2$, $\frac{dS}{dt} = -1\,cm^2/min$, $\frac{dD}{dt} = ?$ at $D = 10\,cm$
(D = Diameter of the snowball).

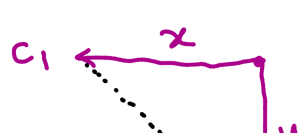
Since $r = \frac{D}{2}$,

$$S = 4\pi\left(\frac{D}{2}\right)^2 = \pi D^2$$

$$\Rightarrow \frac{dS}{dt} = 2\pi D\frac{dD}{dt} \Rightarrow \frac{dD}{dt} = \frac{1}{2\pi D}\frac{dS}{dt}$$

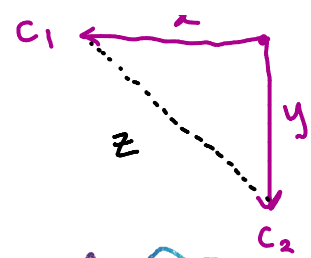
$$= \frac{1}{2\pi(10)}\cdot(-1) = -\frac{1}{20\pi}$$

Hence, the diameter of the snowball decreases at the rate of $\frac{1}{20\pi}\,cm/min$.

(17) 

$\frac{dx}{dt} = 60\,mi/h$, $\frac{dy}{dt} = 25\,mi/h$, $\frac{dz}{dt} = ?$ at $t = 2$.

(17)



$\frac{dx}{dt} = 60$ mi/h, $\frac{dy}{dt}$ = [illegible], $\frac{dz}{dt}$ [illegible]

At $t = 2h$,

$x = 60(2) = 120$ mi, $y = 25(2) = 50$ mi

$\frac{dx}{dt} = 60 \Rightarrow x(t) = 60t$

$\frac{dy}{dt} = 25 \Rightarrow y(t) = 25t$

By Pythagoras' rule,

$z^2 = x^2 + y^2 \Rightarrow z^2 = 120^2 + 50^2 \Rightarrow z = 130$ mi

$\Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$$\Rightarrow \frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt} + \frac{y}{z}\frac{dy}{dt}$$

$$= \frac{120}{130}(60) + \frac{50}{130}(25)$$

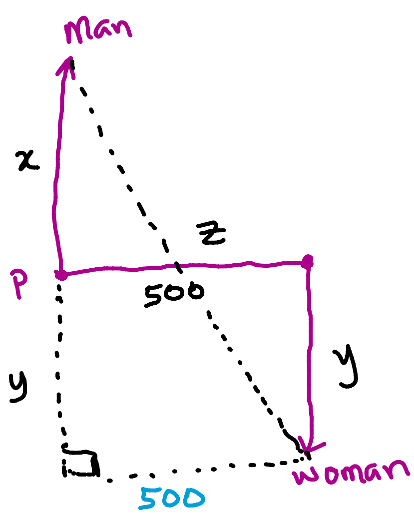
$$= \frac{120(60) + 50(25)}{130}$$

$$= \frac{8450}{130}$$

$$= 65$$

Hence, the distance between the two cars is increasing at the rate of 65 mi/h

(19)



x = distance man walks

y = distance woman walks

z = their distance apart

$\frac{dx}{dt} = 4$ ft/s, $\frac{dy}{dt} = 5$ ft/s

$\Rightarrow x(t) = 4t$ and $y(t) = 5t$

After $t = 15$ min $= 15(60)$s $= 900$s,

woman is $y = 5(900) = 4500$ ft from P.

and the man being 5 minutes earlier is

$x = 4(900 + 5 \times 60) = 4800$ ft from P.

By Pythagoras rule,

$z^2 = (x+y)^2 + 500^2$

$= x^2 + 2xy + y^2 + 250000$

z = their distance apart

$$z^2 = (x+y)^2 + 500$$
$$= x^2 + 2xy + y^2 + 250000$$
$$= 4800^2 + 2(4800)(4500) + 4500^2 + 250000$$
$$\doteq 86740000$$
$$\Rightarrow z = \sqrt{86740000}$$

$$\Rightarrow \frac{d}{dt}(z^2) = \frac{d}{dt}\left(x^2 + 2xy + y^2 + 250000\right)$$

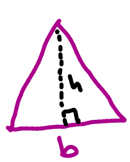
$$\Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2x\frac{dy}{dt} + 2y\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\Rightarrow \frac{dz}{dt} = \left(\frac{x+y}{z}\right)\frac{dx}{dt} + \left(\frac{x+y}{z}\right)\frac{dy}{dt}$$
$$= \left(\frac{x+y}{z}\right)\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$
$$= \frac{4800+4500}{\sqrt{86740000}}(4+5)$$
$$= \frac{83700}{\sqrt{86740000}} \simeq 8.987$$

Hence the people are moving apart at the rate of approximately 8.987ft/s.

(21) Area of triangle $A = \frac{1}{2}bh$, $\frac{dh}{dt} = 1\text{cm/min}$, $\frac{dA}{dt} = 2\text{cm}^2/\text{min}$, $\frac{db}{dt} = ?$

at $h = 10\text{cm}$ and $A = 100\text{cm}^2$.



$h = 10\text{cm}$ and $A = 100\text{cm}^2 \Rightarrow 100 = \frac{1}{2}(b)(10) = 5b \Rightarrow b = 20\text{cm}$

So

$$\frac{dA}{dt} = \frac{d}{dt}\left(\frac{1}{2}bh\right) = \frac{1}{2}b\frac{dh}{dt} + \frac{1}{2}h\frac{db}{dt}$$

$$\Rightarrow \quad \frac{1}{2}h\frac{db}{dt} = \frac{dA}{dt} - \frac{1}{2}b\frac{dh}{dt}$$

$$\Rightarrow \quad \frac{db}{dt} = \frac{2}{h}\frac{dA}{dt} - \frac{b}{h}\frac{dh}{dt}$$

$$= \frac{2}{10}(2) - \frac{20}{10}(1)$$

$$= \frac{4-20}{10}$$

$$= \frac{-16}{10}$$

$$= -1.6$$

Hence, the base of the triangle is decreasing at the rate of 1.6 cm/min.