Lecture Note 19 (Ref. text book page 245)

19/01/2021

3.9 Related Rates

If we are pumping air into a balloon, both the volume and the radius of the balloon are increasing and their rates of increase are related to each other. But it is much easier to measure directly the rate of increase of the volume than the rate of increase of the radius.

In a related rates problem the idea is to compute the rate of change of one quantity in terms of the rate of change of another quantity (which may be more easily measured). The procedure is to find an equation that relates the two quantities and then use the Chain Rule to differentiate both sides with respect to time.

Example 1 If V is the volume of a cube with edge length x and the cube expands as time passes, find dV/dt in terms of dx/dt.

Example 2 The radius of a sphere is increasing at a rate of 4mm/s. How fast is the volume increasing when the dimater is 80mm?

Example 3 Suppose $y = \sqrt{2x+1}$, where x and y are functions of t.

(a) if $dx/dt = 3$, find dy/dt when $x = 4$.

(b) if $dy/dt = 5$, find dx/dt when $x = 12$.

Example 4 Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

Problem Solving Strategy

1. Read the problem carefully.
2. Draw a diagram if possible.
3. Introduce notation. Assign symbols to all quantities that are functions of time.
4. Express the given information and the required rate in terms of derivatives.
5. Write an equation that relates the various quantities of the problem. If necessary, use the geometry of the situation to eliminate one of the variables by substitution.
6. Use the Chain Rule to differentiate both sides of the equation with respect to t.
7. Substitute the given information into the resulting equation and solve for the unknown rate.

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