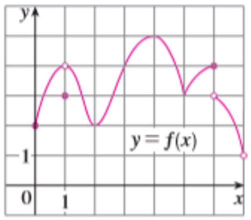
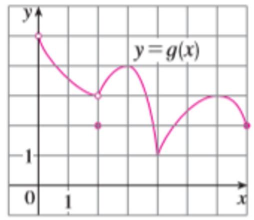
5 **5–6** Use the graph to state the absolute and local maximum and minimum values of the function.

5.



6.



Local maximum values are

$f(4)=5$ and $f(6)=4$

So absolute maximum value is

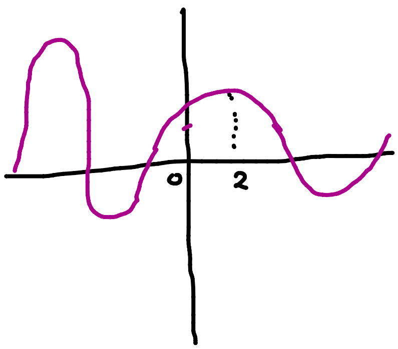
$f(4) = 5$.

Local minimum values are

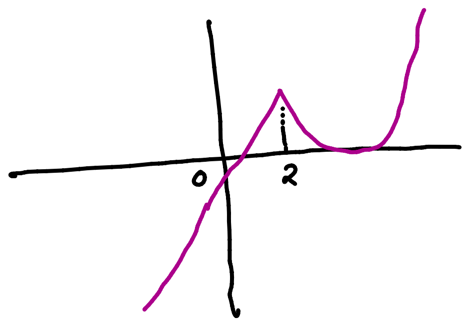
$f(0)=2=f(2)$ and $f(1)=3=f(5)$

Here, there is no absolute minimum.

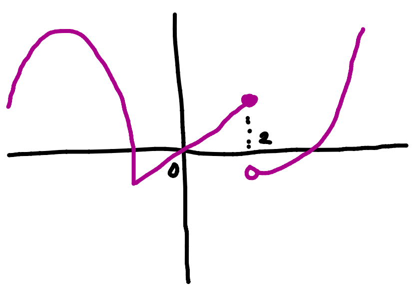
11 (a)




(b)




(c)




30 $f(x) = x^3 + 6x^2 - 15x$

$\Rightarrow f'(x) = 3x^2 + 12x - 15 = 0$

$\Rightarrow x^2 + 4x - 5 = 0$

$\Rightarrow (x+5)(x-1) = 0$

$\Rightarrow x = -5, 1$ are the required critical numbers.

$g(x) = x-1$

(35) $g(y) = \dfrac{y-1}{y^2-y+1}$

$$\Rightarrow g'(y) = \frac{(y^2-y+1)(1) - (y-1)(2y-1)}{(y^2-y+1)^2} = 0$$

$$\Rightarrow y^2-y+1-(2y^2-y-2y+1) = 0$$

$$\Rightarrow y^2-y+1-2y^2+y+2y-1 = 0$$

$$\Rightarrow -y^2+2y = 0$$

$$\Rightarrow -y(y-2) = 0$$

$\Rightarrow y = 0, 2$ are the critical numbers.

(39) $F(x) = x^{4/5}(x-4)^2$

$$\Rightarrow F'(x) = \frac{4}{5}x^{-1/5}(x-4)^2 + 2x^{4/5}(x-4) = 0$$

$$\Rightarrow (x-4)\left(\frac{4}{5}x^{-1/5}(x-4) + 2x^{4/5}\right) = 0$$

$$\Rightarrow (x-4)\left(\frac{4}{5}x^{4/5} - \frac{16}{5}x^{-1/5} + 2x^{4/5}\right) = 0$$

$$\Rightarrow (x-4)\left(\frac{14}{5}x^{4/5} - \frac{16}{5}x^{-1/5}\right) = 0$$

$$\Rightarrow (x-4)x^{-1/5}\left(\frac{14}{5}x - \frac{16}{5}\right) = 0$$

$$\Rightarrow \frac{(x-4)(14x-16)}{x^{1/5}} = 0$$

So $F'(x) = 0 \Rightarrow x = \frac{8}{7}, 4$ and $F'(0)$ DNE. Hence, $0, \frac{8}{7}, 4$ are the critical numbers of $F$.

So $f'(x) = 0 \Rightarrow x = \frac{[illegible]}{7}$, [illegible]
the critical numbers of F.

(41) $f(\theta) = 2\cos\theta + \sin^2\theta$

$$\Rightarrow f'(\theta) = -2\sin\theta + 2\cos\theta\sin\theta = 0$$

$$\Rightarrow \sin\theta(-1+\cos\theta) = 0$$

$$\Rightarrow \sin\theta = 0 \text{ or } \cos\theta = 1$$

$$\Rightarrow \theta = \pi n \text{ or } \theta = 2\pi n, \text{ n an integer.}$$

Hence, critical numbers are $\{\pi n\}_{n\in\mathbb{Z}} = \{\ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots\}$.

(49) $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$.

$$f'(x) = 6x^2 - 6x - 12 = 0$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x-2)(x+1) = 0$$

$\Rightarrow x = -1, 2$ are the critical numbers.

Thus,

$$f(-2) = 2(-2)^3 - 3(-2)^2 - 12(-2) + 1 = -3$$
$$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1 = 8$$
$$f(2) = 2(2^3) - 3(2^2) - 12(2) + 1 = -19$$
$$f(3) = 2(3^3) - 3(3^2) - 12(3) + 1 = -8.$$

Hence, $f(2) = -19$ is the absolute minimum while $f(-1) = 8$ is the absolute maximum.

(50) $f(x) = x^3 - 6x^2 + 5$, $[-3, 5]$.

(50) $f(x) = x^3 - 6x^2 + 5,\ [-3, 5]$.

$f'(x) = 3x^2 - 12x = 0$

$\Rightarrow 3x(x-4) = 0$

$\Rightarrow x = 0,\ x = 4$ (critical numbers)

Thus,

$f(-3) = (-3)^3 - 6(-3)^2 + 5 = -76$

$f(0) = (0)^3 - 6(0)^2 + 5 = 5$

$f(4) = 4^3 - 6(4^2) + 5 = -27$

$f(5) = 5^3 - 6(5^2) + 5 = -20$

Hence, $f(-3) = -76$ is the absolute minimum value while $f(0) = 5$ is the absolute maximum value.

(53) $f(x) = x + \frac{1}{x},\ [0.2, 4]$.

$= x + x^{-1}$

$\Rightarrow f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2} = 0$

$\Rightarrow (x-1)(x+1) = 0$ and $f'(0) = -\infty$

$\Rightarrow$ $x = 1$ is the critical number since 0 and $-1$ are not in $[0.2, 4]$.

$f(1) = 1 + \frac{1}{1} = 2$

$f(0.2) = 0.2 + \frac{1}{0.2} = 5.2$

$f(4) = 4 + \frac{1}{4} = 4.25$

Hence, $f(1) = 2$ is the absolute minimum while $f(0.2) = 5.2$

... maximum.

Hence, $f(1) = 2$ is the absolute ...

is the absolute maximum.

(59) $f(x) = x^{-2}\ln x, \ [\tfrac{1}{2}, 4].$

$$f'(x) = x^{-2}\left(\frac{1}{x}\right) - 2x^{-3}\ln x$$

$$= \frac{1}{x^3} - \frac{2\ln x}{x^3}$$

$$= \frac{1}{x^3}(1 - 2\ln x) = 0$$

$\Rightarrow f'(0) = \infty$ and $1 - 2\ln x = 0$

$\Rightarrow f'(0) = \infty$ and $\ln x = \frac{1}{2}$

$\Rightarrow f'(0) = \infty$ and $x = e^{1/2}$

$\Rightarrow x = e^{1/2}$ is the critical number since 0 is not in $[1/2, 4]$.

Thus,

$$f(\tfrac{1}{2}) = (\tfrac{1}{2})^{-2}\ln(\tfrac{1}{2}) = 4\ln(\tfrac{1}{2}) \approx -2.7728$$

$$f(e^{1/2}) = (e^{1/2})^{-2}\ln(e^{1/2}) = \frac{1}{2e} \approx 0.1839$$

$$f(4) = 4^{-2}\ln 4 \approx 0.08664$$

Hence, $f(\frac{1}{2}) = 4\ln(\frac{1}{2})$ is the absolute minimum while

$f(e^{1/2}) = \frac{1}{2e}$ is the absolute maximum.

(60) $f(x) = xe^{x/2}, \ [-3, 1].$

$f'(x) = x\left(\frac{1}{2}e^{x/2}\right) + e^{x/2} = 0$

$$f'(x) = x\left(\tfrac{1}{2}e^{x/2}\right) + e^{x/2} = 0$$

$$\Rightarrow \quad e^{x/2}\left(\frac{x}{2}+1\right) = 0$$

$$\Rightarrow \quad \frac{x}{2}+1 = 0$$ since there is no $x$ such that $e^{x/2} = 0$.

$$\Rightarrow \quad x = -2$$ is the critical number.

Thus,

$$f(-3) = (-3)e^{-3/2} \approx -0.6694$$

$$f(-2) = -2e^{-2/2} = -2e^{-1} = \frac{-2}{e} \approx -0.7358$$

$$f(1) = 1e^{1/2} = e^{1/2} \approx 1.6487$$

Hence, $f(-2) = \frac{-2}{e}$ is the absolute minimum while $f(1) = e^{1/2}$ is the absolute maximum.