Lecture Note 21 (Ref. text book page 276) 10/12/2021

CHAPTER 4

APPLICATION OF DIFFERENTIATION

4.1 Maximum and Minimum Values

Definition 1. Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute maximum** value of f on D if $f(c) \geq f(x)$ for all x in D.
- **absolute minimum** value of f on D if $f(c) \leq f(x)$ for all x in D.

2. The number $f(c)$ is a

- **local maximum** value of f if $f(c) \geq f(x)$ when x is near c.
- **local minimum** value of f if $f(c) \leq f(x)$ when x is near c.

The maximum and minimum values of f are called **extreme values** of f.

Question: Does a function always have an extreme value?

The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Question: What is the relation between a local minimum with the derivative?

Fermat's Theorem If f has a local maximum or minimum at c, and if $f^{'}(c)$ exists, then $f^{'}(c) = 0$.

Definition A **critical number** of a function f is a number c in the domain of f such that either $f^{'}(c) = 0$ or $f^{'}(c)$ does not exist.
Therefore if f has a local maximum or minimum at c, then c is a critical number of f.

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Example 1 (a) Find all critical numbers of f.
(b) Find the absolute maximum and absolute minimum values of f on the given interval.

(i) $f(x) = x^3 - 6x^2 + 5, \quad [-3, 5]$

(ii) $f(x) = x + 1/x, \quad [0.2, 4]$

(iii) $f(t) = (t^2 - 4)^3, \quad [-2, 3]$

(iv) $f(t) = 2\cos t + \sin 2t, \quad [0, \pi/2]$

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