(11) $f(x) = 2x^2 - 3x + 1$, $[0, 2]$.

Since f is a polynomial:

① f is continuous everywhere and so continuous on $[0, 2]$.

② f is differentiable everywhere and so differentiable on $(0, 2)$.

So by Mean Value Theorem, we can find c in $(0, 2)$ such that

$$f'(c) = \frac{f(2) - f(0)}{2 - 0}$$

$$\Rightarrow \quad 4c - 3 = \frac{2(2^2) - 3(2) + 1 - 1}{2} = 1 \text{ in } (0, 2).$$

$$\Rightarrow \quad 4c = 1 + 3 = 4 \quad \Rightarrow \quad c = 1 \text{ in}$$

(14) $f(x) = \frac{1}{x}$, $[1, 3]$.

Domain$(f) = (-\infty, 0) \cup (0, \infty)$ and f is continuous and differentiable on it. So f is continuous on $[1, 3]$ and differentiable on $(1, 3)$. Thus, by Mean Value Theorem, we can find c in $(1, 3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \quad -\frac{1}{c^2} = \frac{\frac{1}{3} - \frac{1}{1}}{3 - 1} = \frac{-2/3}{2} = -\frac{1}{3}$$

$$\Rightarrow \quad c^2 = 3 \quad \Rightarrow \quad c = \pm\sqrt{3} \quad \Rightarrow \quad c = \sqrt{3} \text{ in } (1, 3).$$

25. $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small is $f(4)$.

By Mean Value Theorem,

$$\frac{f(4) - f(1)}{4 - 1} = f'(c) \quad \text{for some } c \text{ in } (1, 4).$$

$$\Rightarrow \quad f(4) - f(1) = 3f'(c)$$

$$\Rightarrow \quad f(4) = 3f'(c) + f(1)$$

$$= 3f'(c) + 10$$

$$\geq 3(2) + 10$$

$$= 16$$

Hence, the smallest $f(4)$ can be is 16.

27. Suppose there is a function f such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \leq 2$ for all x, then by Mean Value Theorem, we can find a c in $(0, 4)$ such that

$$f'(c) = \frac{f(2) - f(0)}{2 - 0} = \frac{4 - (-1)}{2} = \frac{5}{2} = 2.5$$

But $f'(c) \leq 2 < 2.5$.

So such c does not exist.