Lecture Note 22 (Ref. text book page 287)   10/13/2021

4.2 The Mean Value Theorem

We will see that many of the results of this chapter depend on one central fact, which is called the Mean Value Theorem. But to arrive at the Mean Value Theorem we first need the following result.

Rolle's Theorem Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$.

2. f is differentiable on the open interval (a, b).

3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.

Example 1 Verify that the function $f(x) = x + 1/x$ satisfies the three hypotheses of Rolle's Theorem on the given interval $[1/2, 2]$. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

The Mean Value Theorem Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a, b]$.

2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{or, equivalently} \quad f(b) - f(a) = f'(c)(b - a)$$

Example 2 Verify that the function $f(x) = x^3 - 3x + 2$ satisfies the hypotheses of Mean Value Theorem on the given interval $[-2, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.