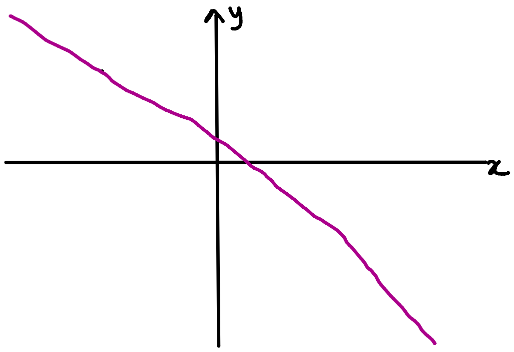
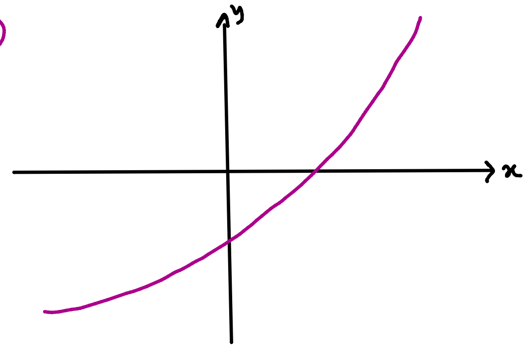
24 (a) $f'(x) < 0$ and $f''(x) < 0$ for all x

(b) $f'(x) > 0$ and $f''(x) > 0$ for all x.

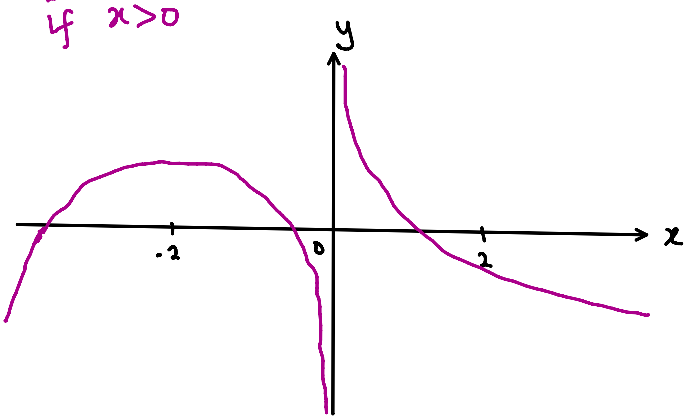
(a)



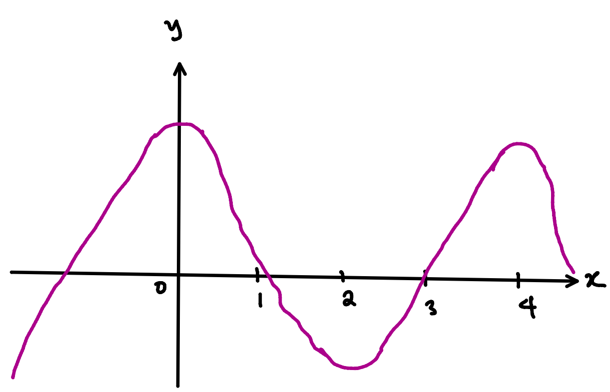
(b)



26 Vertical asymptote $x = 0$, $f'(x) > 0$ if $x < -2$,
$f'(x) < 0$ if $x > -2$ $(x \neq 0)$, $f''(x) < 0$ if $x < 0$,
$f''(x) > 0$ if $x > 0$



27 $f'(0) = f'(2) = f'(4) = 0$,
$f'(x) > 0$ if $x < 0$ or $2 < x < 4$,
$f'(x) < 0$ if $0 < x < 2$ or $x > 4$
$f''(x) > 0$ if $1 < x < 3$,
$f''(x) < 0$ if $x < 1$ or $x > 3$.



(37) $f(x) = x^3 - 12x + 2$.

$f'(x) = 3x^2 - 12 = 3(x^2 - 4)$
$= 3(x-2)(x+2)$

So
$f'(x) = 0 \Rightarrow x = -2, 2$

	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
$x-2$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$
$f'(x)$	$+$	$-$	$+$
	$f\nearrow$	$f\searrow$	$f\nearrow$

$f''(x) = 6x = 0 \Rightarrow x = 0$

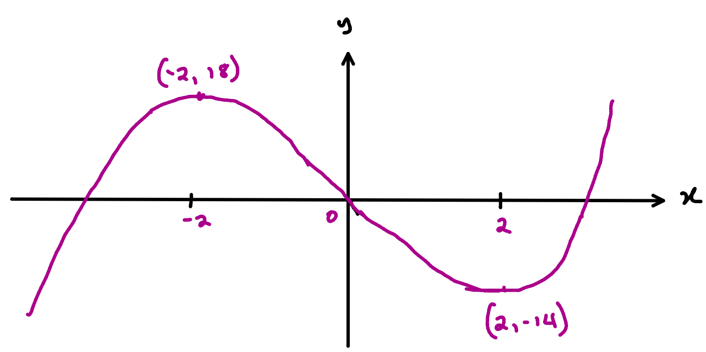
	$(-\infty, 0)$	$(0, \infty)$
x	$-$	$+$
$f''(x)$	$-$	$+$
	CD	CU

(a) f is increasing on $(-\infty, -2)$ and $(2, \infty)$.
f is decreasing on $(-2, 2)$.

(b) $f'(x) = 0 \Rightarrow x = -2, 2$ and from the first table,
$f(-2) = 18$ is a local maximum,
$f(2) = -14$ is a local minimum.

From the second table, f is concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$.
Recall f is concave down when $f''(x) < 0$ and concave up when $f''(x) > 0$. So $(0, 0)$ is an inflection point.

d



(42) $h(x) = 5x^3 - 3x^5$

-1 1

$h'(x) = 15x^2 - 15x^4$

(42) $h(x) = 5x^3 - 3x^5$

$$h'(x) = 15x^2 - 15x^4$$
$$= 15x^2(1-x^2)$$
$$= 15(1-x)(1+x)$$

So $h'(x) = 0 \Rightarrow x = -1, 1.$

	$(-\infty,-1)$	-1	$(-1,1)$	1	$(1,\infty)$
$(1-x)$	$+$		$+$		$-$
$(1+x)$	$-$		$+$		$+$
$h'(x)$	$-$		$+$		$-$
	$h(x) \searrow$		$h(x) \nearrow$		$h(x) \searrow$

From the table, h is decreasing on $(-\infty,-1)$ and $(1,\infty)$ and increasing on $(-1,1)$.

b From the table, $h(-1) = -2$ is a local minimum and $h(1) = 2$ is a local maximum.

$$h'(x) = 15x^2 - 15x^4 \Rightarrow h''(x) = 30x - 60x^3$$
$$= 30x(1-2x^2)$$
$$= 30x(1-\sqrt{2}x)(1+\sqrt{2}x)$$

So $h''(x) = 0 \Rightarrow x = 0$ or $x = \pm\frac{1}{\sqrt{2}} \simeq \pm 0.7071$

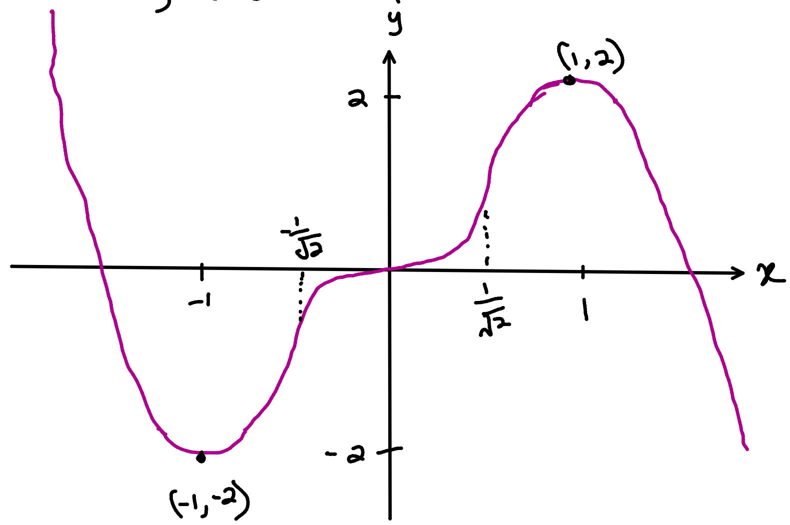
		$-\frac{1}{\sqrt{2}}$		0		$\frac{1}{\sqrt{2}}$	
x	$-$		$-$		$+$		$+$
$1-\sqrt{2}x$	$+$		$+$		$+$		$-$
$1+\sqrt{2}x$	$-$		$+$		$+$		$+$
$h''(x)$	$+$		$-$		$+$		$-$
	CU		CD		CU		CD

Thus,

$\left(-\frac{1}{\sqrt{2}}, h\left(-\frac{1}{\sqrt{2}}\right)\right)$, $(0, h(0))$ and $\left(\frac{1}{\sqrt{2}}, h\left(\frac{1}{\sqrt{2}}\right)\right)$ since

Thus,

inflection points are $\left(-\frac{1}{\sqrt{2}}, h\left(-\frac{1}{\sqrt{2}}\right)\right)$, $(0, h(0))$ and $\left(\frac{1}{\sqrt{2}}, h\left(\frac{1}{\sqrt{2}}\right)\right)$ since h changes concavity at these points.



(43) $F(x) = x\sqrt{6-x}$. Notice that domain of f is $(-\infty, 6]$.

$$F'(x) = -\frac{x}{2}(6-x)^{-1/2} + (6-x)^{1/2}$$

$$= \frac{-x}{2\sqrt{6-x}} + \sqrt{6-x}$$

$$= \frac{-x + 2(6-x)}{2\sqrt{6-x}}$$

$$= \frac{12-3x}{2\sqrt{6-x}}$$

So $F'(x) = 0 \Rightarrow \frac{12-3x}{2\sqrt{6-x}} = 0 \Rightarrow x = 4.$

	$(-\infty, 4)$	4	$(4, 6)$	6
$4-x$	$+$		$-$	
$\sqrt{6-x}$	$+$		$+$	
$F'(x)$	$+$		$-$	
	$F \nearrow$		$F \searrow$	

So F is increasing on $(-\infty, 4)$ and decreasing on $(4, 6)$.

From the table, $F(4) = 4\sqrt{2}$ is a local maximum value.

$$F'(x) = \frac{12-3x}{2\sqrt{6-x}}$$

$$\Rightarrow F''(x) = \frac{2\sqrt{6-x}\,(-3) - (12-3x)\left(-\frac{1}{\sqrt{6-x}}\right)}{4(6-x)}$$

$$= \frac{-6\sqrt{6-x} + \frac{12-3x}{\sqrt{6-x}}}{4(6-x)}$$

$$= \frac{-6(6-x) + 12-3x}{4(6-x)^{3/2}}$$

$$= \frac{-36+6x+12-3x}{4(6-x)^{3/2}}$$

$$= \frac{3(x-8)}{4(6-x)^{3/2}}$$

$$= \frac{3(x-8)}{4(6-x)^{3/2}}$$

	6		8	
$x-8$	$-$	$-$		$+$
$(6-x)^{3/2}$	$+$	DNE		DNE
$f''(x)$	$-$	DNE		DNE
	CD			

So f is concave down in $(-\infty, 6)$.

(46) $f(x) = \ln(x^2+9)$. Notice that $D(f) = (-\infty, \infty)$.

$$f'(x) = \frac{2x}{x^2+9} = 0 \Rightarrow x = 0$$

	0	
x	$-$	$+$
x^2+9	$+$	$+$
$f'(x)$	$-$	$+$
	$f \searrow$	$f \nearrow$

So f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

b From the table, $f(0) = \ln 9$ is a local minimum.

$$f'(x) = \frac{2x}{x^2+9}$$

$$f'(x) = \frac{2x}{x^2+9}$$

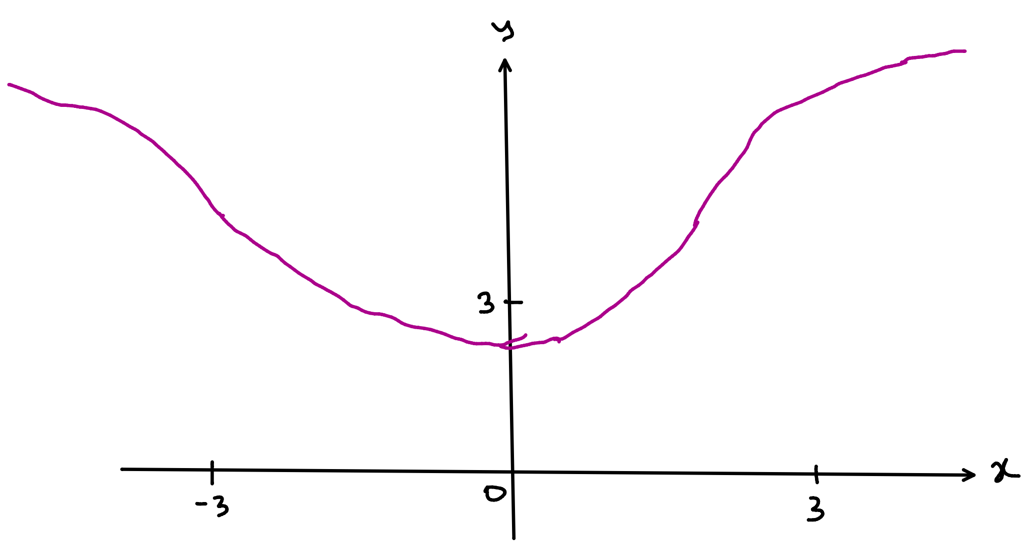
$$\Rightarrow f''(x) = \frac{(x^2+9)(2) - 2x(2x)}{(x^2+9)^2} = \frac{2x^2 + 18 - 4x^2}{(x^2+9)^2}$$

$$= \frac{18 - 2x^2}{(x^2+9)^2}$$

$$= \frac{2(3-x)(3+x)}{(x^2+9)^2}$$

So $f''(x) = 0 \Rightarrow x = -3, 3.$

		-3		3	
$3-x$	$+$		$+$		$-$
$3+x$	$-$		$+$		$+$
$(x^2+9)^2$	$+$		$+$		$+$
$f''(x)$	$-$		$+$		$-$
	CD		CU		CD



(50) $f(x) = \dfrac{x^2-4}{x^2+4} = \dfrac{(x-2)(x+2)}{x^2+4}$

a Since the denominator $x^2+4>0$ for all x, there is no vertical asymptote.

On the other hand,

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x^2-4}{x^2+4} = \lim_{x\to\pm\infty} \frac{1-\frac{4}{x^2}}{1+\frac{4}{x^2}} = \frac{1-0}{1+0} = 1$$

$\Rightarrow$ $y=1$ is a horizontal asymptote.

$$f'(x) = \frac{(x^2+4)(2x)-(x^2-4)(2x)}{(x^2+4)^2} = \frac{2x^3+8x-2x^3+8x}{(x^2+4)^2}$$

$$= \frac{16x}{(x^2+4)^2}$$

So $f'(x)=0 \Rightarrow \dfrac{16x}{(x^2+4)^2}=0 \Rightarrow x=0$

		0	
$16x$	$-$		$+$
$(x^2+4)^2$	$+$		$+$
$f'(x)$	$-$		$+$
	$f \searrow$		$f \nearrow$

Thus, f is increasing on $(0,\infty)$ and decreasing on $(-\infty,0)$.

f changes from decreasing to increasing at $x=0$. So

$f(0) = \dfrac{0^2-4}{0^2+4} = -1$ is a local minimum value.

d $f'(x) = \dfrac{16x}{(x^2+4)^2}$

d $\quad f'(x) = \dfrac{16x}{(x^2+4)^2}$

$$\Rightarrow f''(x) = \frac{(x^2+4)^2 16 - 16x\left(4x(x^2+4)\right)}{(x^2+4)^4}$$

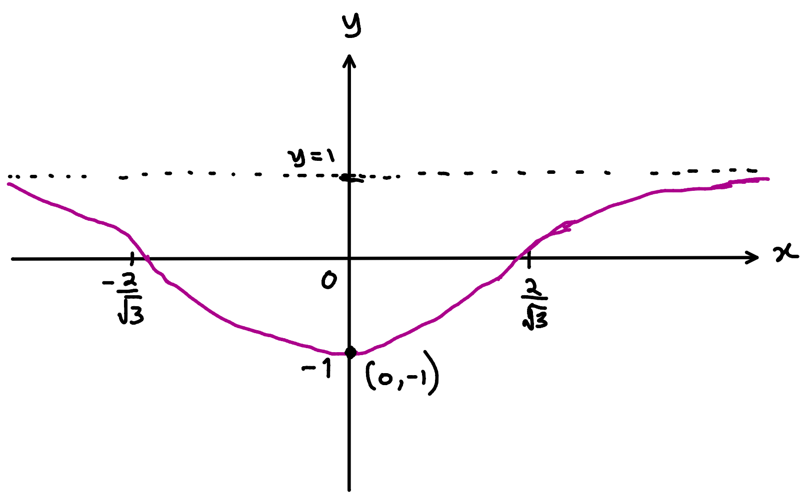
$$= \frac{16(x^2+4)\left(x^2+4-4x^2\right)}{(x^2+4)^4}$$

$$= \frac{16(4-3x^2)}{(x^2+4)^3}$$

$\frac{2}{\sqrt{3}} \approx 1.1547$

So
$f''(x) = 0 \Rightarrow x = \pm \frac{2}{\sqrt{3}}$

		$-\frac{2}{\sqrt{3}}$		$\frac{2}{\sqrt{3}}$	
$16(4-3x^2)$	$-$		$+$		$-$
$(x^2+4)^3$	$+$		$+$		$+$
$f''(x)$	$-$		$+$		$-$
	CD		CU		CD



(51) $f(x) = \sqrt{x^2+1} - x$

There is no vertical asymptotes since $D(f) = (-\infty, \infty)$.

(51) $f(x) =$

There is no vertical asymptotes since $D(f) = (-\infty, \infty)$.

For horizontal asymptotes,

$$\lim_{x \to -\infty} \left(\sqrt{x^2+1} - x\right) = \infty - (-\infty) = \infty$$

and

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left(\sqrt{x^2+1} - x\right)$$

$$= \lim_{x \to \infty} \frac{\left(\sqrt{x^2+1} - x\right)\left(\sqrt{x^2+1} + x\right)}{\sqrt{x^2+1} + x}$$

$$= \lim_{x \to \infty} \frac{x^2+1 - x^2}{\sqrt{x^2+1} + x}$$

$$= \lim_{x \to \infty} \frac{1}{x\sqrt{1+\frac{1}{x^2}} + x}$$

$$= \lim_{x \to \infty} \frac{1/x}{\sqrt{1+1/x^2} + 1}$$

$$= \frac{0}{2} = 0$$

So $y = 0$ is a horizontal asymptote.

b $f'(x) = x\left(x^2+1\right)^{-1/2} - 1 = \dfrac{x}{\sqrt{x^2+1}} - 1 \leq 1 - 1 = 0$ since b $f'(x) = x(x^2+1)^{-1/2} - 1 = \frac{x}{\sqrt{x^2+1}} - 1 \leq 1 - 1 = 0$ since

$$\frac{x}{\sqrt{x^2+1}} < 1 \quad \text{for all } x.$$

So f is decreasing on $(-\infty, \infty)$.

No minimum or maximum since $f'(x) = 0$ has no solution.
Moreover, f is decreasing on the whole of $\mathbb{R}$.

d $f'(x) = \frac{x}{\sqrt{x^2+1}} - 1 = \frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}}$

$$\Rightarrow f''(x) = \frac{\sqrt{x^2+1}\left(1 - \frac{x}{\sqrt{x^2+1}}\right) - \left(x - \sqrt{x^2+1}\right)\left(\frac{x}{\sqrt{x^2+1}}\right)}{x^2+1}$$

$$= \frac{\sqrt{x^2+1}\left(\frac{\sqrt{x^2+1} - x}{\sqrt{x^2+1}}\right) - \left(x - \sqrt{x^2+1}\right)\left(\frac{x}{\sqrt{x^2+1}}\right)}{x^2+1}$$

$$= \frac{\left(\frac{\sqrt{x^2+1} - x}{\sqrt{x^2+1}}\right)\left(\sqrt{x^2+1} + 1\right)}{x^2+1}$$

$$= \frac{\left(1 - \frac{x}{\sqrt{x^2+1}}\right)\left(\sqrt{x^2+1} + 1\right)}{x^2+1} > 0 \text{ for all } x.$$

$$= \frac{(\quad)}{x^2+1}$$

So f is CU on $(-\infty, \infty)$. There is no inflection point.