Reminder: $\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$, $\infty-\infty$, $0\cdot(\pm\infty)$ $\left(\& \; 0^0, \infty^0, 1^\infty\right)$

Given
$\lim_{x\to a} f(x)=0$, $\lim_{x\to a} g(x)=0$, $\lim_{x\to a} h(x)=1$, $\lim_{x\to a} p(x)=\infty$, $\lim_{x\to a} q(x)=\infty$.

1 ⓐ $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ is indeterminate form.

ⓑ $\lim_{x\to a}\frac{f(x)}{p(x)} = \frac{0}{\infty} = 0.$

ⓒ $\lim_{x\to a}\frac{h(x)}{p(x)} = \frac{1}{\infty} = 0$

ⓓ $\lim_{x\to a}\frac{p(x)}{f(x)} = \begin{cases} \infty & \text{if } f(x)>0 \text{ for } x \text{ close to } a \\ -\infty & \text{if } f(x)<0 \text{ for } x \text{ close to } a. \end{cases}$

ⓔ $\lim_{x\to a}\frac{p(x)}{q(x)} = \frac{\infty}{\infty}$ is indeterminate form

2 a $\lim_{x\to a}[f(x)p(x)] = 0\cdot\infty$ is indeterminate form

ⓑ $\lim_{x\to a}[h(x)p(x)] = 1\cdot\infty = \infty$

ⓒ $\lim_{x\to a}[p(x)q(x)] = \infty\cdot\infty = \infty.$

3 $\lim_{x\to a}[f(x)-p(x)] = 0-\infty = -\infty$

b $\lim [p(x)-q(x)] = \infty-\infty$ is indeterminate b $\lim_{x \to a} [p(x) - q(x)] = \infty - \infty$ is indeterminate

c $\lim_{x \to a} [p(x) + q(x)] = \infty + \infty = \infty$

4 $\lim_{x \to a} [f(x)]^{g(x)} = 0^0$ is indeterminate form

b $\lim_{x \to a} [f(x)]^{p(x)} = 0^\infty = 0$

$\lim_{x \to a} [h(x)]^{p(x)} = 1^\infty$ is indeterminate form

8 $\lim_{x \to 3} \frac{x-3}{x^2-9} = \lim_{x \to 3} \frac{x-3}{(x-3)(x+3)} = \lim_{x \to 3} \frac{1}{x+3} = \frac{1}{3+3} = \frac{1}{6}$

(Notice that L'Hospital's rule works also).

15 $\lim_{t \to 0} \frac{e^{2t}-1}{\sin t} = \frac{0}{0}$. So $\lim_{t \to 0} \frac{e^{2t}-1}{\sin t} \overset{H}{=} \lim_{t \to 0} \frac{2e^{2t}}{\cos t} = 2$.

25 $\lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1-4x}}{x} = \frac{0}{0}$, so l'Hospital's rule will work

but multiplying top and bottom by conjugate of the top might be easier than differentiating for l'Hospital's rule.

So

$$\lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1-4x}}{x} = \lim_{x \to 0} \frac{\left(\sqrt{1+2x} - \sqrt{1-4x}\right)\left(\sqrt{1+2x} + \sqrt{1-4x}\right)}{x\left(\sqrt{1+2x} + \sqrt{1-4x}\right)}$$

$$= \lim_{x\to 0} \frac{(1+2x)-(1-4x)}{x\left(\sqrt{1+2x}+\sqrt{1-4x}\right)}$$

$$= \lim_{x\to 0} \frac{6}{\sqrt{1+2x}+\sqrt{1-4x}}$$

$$= \frac{6}{1+1} = 3.$$

Use l'Hospital's rule to confirm the answer!

(33) $\lim_{x\to 0} \frac{x(3^x)}{3^x-1} = \frac{0}{0}$

Recall $y = 3^x \Rightarrow \ln y = \ln 3^x = x\ln 3$

$\Rightarrow \frac{d}{dx}\ln y = \frac{d}{dx}(x\ln 3)$

$\Rightarrow \frac{y'}{y} = \ln 3$

$\Rightarrow y' = y\ln 3$

$= \underline{\underline{3^x \ln 3}}$

So

$$\lim_{x\to 0} \frac{x(3^x)}{3^x-1} \overset{H}{=} \lim_{x\to 0} \frac{x(3^x)\ln 3 + 3^x}{3^x \ln 3}$$

$$= \frac{1}{\ln 3}$$

(43) $\lim_{x\to\infty} x\sin(\pi/x) = \infty\cdot 0$

So

$$\lim_{x\to\infty} x\sin(\pi/x) = \lim_{x\to\infty} \frac{\sin(\pi/x)}{1/x}$$

Recall $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$= \lim_{x\to\infty} \frac{\pi\sin(\pi/x)}{\pi/x}$$

$$= \pi \lim \frac{\sin(\pi/x)}{}$$

$$= \pi \lim_{x\to\infty} \frac{\sin(\pi/x)}{\pi/x}$$

$$= \pi \lim_{y\to 0} \frac{\sin y}{y}, \quad y = \pi/x \to 0 \text{ as } x \to \infty.$$

$$= \pi(1)$$

$$= \underline{\underline{\pi}}$$ (Use l'Hospital's rule to check the answer).

(44) $\lim_{x\to\infty} \sqrt{x}\, e^{-x/2} = \infty \cdot 0$

So

$$\lim_{x\to\infty} \sqrt{x}\, e^{-x/2} = \lim_{x\to\infty} \frac{\sqrt{x}}{e^{x/2}} \overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{2}e^{x/2}}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{x}\, e^{x/2}}$$

$$= 0$$

(51) $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right) = \infty - \infty$

So

$$\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right) = \lim_{x\to 1} \frac{x\ln x - x + 1}{(x-1)\ln x}$$

$$\overset{H}{=} \lim_{x\to 1} \frac{x\left(\frac{1}{x}\right) + \ln x - 1}{(x-1)\left(\frac{1}{x}\right) + \ln x}$$

$$= \lim_{x\to 1} \frac{\ln x}{\frac{x-1+x\ln x}{x}}$$

$$= \lim_{x\to 1} \frac{x\ln x}{x-1+x\ln x} = \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 1} \frac{x\left(\frac{1}{x}\right) + \ln x}{1 + x\left(\frac{1}{x}\right) + \ln x}$$

$$= \lim_{x\to 1} \frac{1+\ln x}{2+\ln x} = \frac{1}{2}$$

53 $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{e^x-1}\right) = \infty - \infty$

So

$$\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{e^x-1}\right) = \lim_{x\to 0^+} \frac{e^x-1-x}{xe^x-x}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{e^x-1}{xe^x+e^x-1} = \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{e^x}{xe^x+e^x+e^x}$$

$$= \frac{1}{0+1+1} = \frac{1}{2}$$

UP NEXT

UP NEXT

GUIDELINES FOR SKETCHING A CURVE

Where possible, locate

(a) Domain

(b) Intercepts

(c) Symmetry (even & odd) and periodic for trigs.

(d) Asymptotes

(e) Local maximum and minimum values

(f) Intervals of Increase (↗) or decrease (↘).

(g) Concavity and Points of Inflection

(H) Sketch the Curve