**Lecture Note 25** (Ref. text book page 315) 10/20/2021

# 4.5 Summary of Curve Sketching

## Guidelines for Sketching a Curve

The following checklist is intended as a guide to sketching a curve $y = f(x)$ by hand. Not every item is relevant to every function. (For instance, a given curve might not have an asymptote). But the guidelines provide all the information you need to make a sketch that displays the most important aspects of the function.

**A. Domain:** It's often useful to start by determining the domain $D$ of $f$, that is, the set of values of $x$ for which $f(x)$ is defined.

**B. Intercepts:** The $y$-intercept is $f(0)$ and this tells us where the curve intersects the $y$-axis. To find $x$-intercepts, we set $y = 0$ and solve for $x$. (You can omit this step if the equation is difficult to solve.)

**C. Symmetry**

i- **Even function** i.e $f(x) = f(-x)$ for all $x \in D$ and the curve is symmerty about $y-axis$

ii- **Odd function** i.e $f(-x) = -f(x)$ for all $x \in D$ and the curve is symmetry about origin

iii- **Periodic function** i.e $f(x+p) = f(x)$ for all $x \in D$ where $p$ is positive constant.

**D. Asymptotes:**

- *Horizontal Asymptotes.* Recall from section 2.6 that if either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$, then the line $y = L$ is a horizontal asymptote of the curve $y = f(x)$. If it turns out that $\lim_{x\to\infty} f(x) = \infty$ (or $-\infty$), then we do not have an asymptote to the right, but this fact is still useful information for sketching the curve.
- *Vertical Asymptotes.* Recall from section 2.2 that the line $x = a$ is a vertical asymptote if at least one of the following statements is ture:

$$(1) \qquad \lim_{x\to a^+} f(x) = \infty \qquad \lim_{x\to a^+} f(x) = -\infty \qquad \lim_{x\to a^-} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = -\infty$$

(For rational functions you can locate the vertical asymptotes by equating the denominator to 0 after canceling any common factors. Buf for other functions this method does not apply.) Furthermore, in sketching the curve it is very useful to know exactly which of the statements in (1) is true. If $f(a)$ is not defined but $a$ is an endpoint of the domain of $f$, then you should compute $\lim_{x\to a^-} f(x)$ or $\lim_{x\to a^+} f(x)$, whether or not this limit is infinite.

**E. Intervals of Increase or Decrease** Use the I/D Test. Compute $f'(x)$ and find the intervals on which $f'(x)$ is positive ($f$ is increasing) and the intervals on which $f'(x)$ is negative ($f$ is decreasing).

**F. Local Maximum and Minimum Values** Find the critical numbers of $f$ [the numbers $c$ where $f'(c) = 0$ or $f'(c)$ does not exist]. Then use the First Derivative Test. If $f'$ changes from positive to negative at a critical number $c$, then $f(c)$ is a local maximum. If $f'$ changes from negative to positive at $c$, then $f(c)$ is a local minimum.

**G. Concavity and Points of Inflection** Compute $f''(x)$ and use the Concavity Test. The curve is concave upward where $f''(x) > 0$ and concave downward where $f''(x) < 0$. Inflection points occur where the direction of concavity changes.

**H. Sketch the Curve** Using the information in items A-F, draw the graph. Sketch the asymptotes as dashed lines. Plot the intercepts, maximum and minimum points, and inflection points. Then make the curve pass through these points, rising and falling according to D, with concavity according to F, and approaching the asymptotes.

**Example 1** Use the guidelines to sketch the following curve.

(a) $y = x(x-4)^3$ (b) $y = \dfrac{x}{\sqrt{x^2+1}}$ (c) $y = (1+e^x)^{-2}$

**Practice Problems:**

**Section 4.5:** 3, 9, 13, 15, 25, 28, 31, 34, 39,48