Lecture Note 26 (Ref. text book page 330) 10/25/2021

4.7 Optimization Problems

Steps In Solving Optimization Problems

1. **Understand the Problem** The first step is to read the problem carefully until it is clearly understood. Ask yourself: What is the unknown? What are the given quantities? What are the given conditions?

2. **Draw a Diagram** In most problems it is useful to draw a diagram and identify the given and required quantities on the diagram.

3. **Introduce Notation** Assign a symbol to the quantity that is to be maximized or minimized (let's call it Q for now). Also select symbols $(a, b, c, ..., x, y)$ for other unknown quantities and label the diagram with their symbols. It may help to use initials as suggestive symbols- for example, A for area, h for height, t for time.

4. Express Q in terms of some of the other symbols from Step 3.

5. If Q has been expressed as a function of more than one variable in Step 4, use the given information to find relationships (in the form of equations) among these variables. Then use these equations to eliminate all but one of the variables in the expression for Q. Thus Q will be expressed as a function of one variable x, say, $Q = f(x)$. Write the domain of this function in the given context.

6. Use the methods of Sections 4.1 and 4.3 to find the absolute maximum and minimum value of f. In particular, if the domain of f is a closed interval, then the Closed Interval Method in Section 4.1 can be used.

Example 1 Find two numbers whose difference is 100 and whose product is a minimum.

Example 2 Find two positive numbers whose product is 100 and whose sum is a minimum.

Example 3 Find the dimensions of a rectangle with perimeter $100m$ whose area is as large as possible.

Example 4 Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

(a) Draw several diagrams to illustrate the situation, some short boxes with large bases and some tall boxes with small bases. Find the volumes of several such boxes. Does it appear that there is a maximum volume? If so, estimate it.

(b) Draw a diagram illustrating the general situation. Introduce notation and label the diagram with your symbols.

(c) Write an expression for the volume.

(d) Use the given information to write an equation that relates the variables.

(e) Use part (d) to write the volume as a function of one variable.

(f) Finish solving the problem and compare the answer with your estimate in part (a).

Example 5 A rectangular storage container with an open top is to have a volume of $10m^3$. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

Example 6 Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

Example 7 Find the point on the line $y = 2x + 3$ that is closest to the origin.

Example 8 The top and bottom margins of a poster are each $6cm$ and the side margins are each $4cm$. If the area of printed material on the poster is fixed at $384cm^2$, find the dimensions of the poster with the smallest area.

Example 9 A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is

(a) a maximum?

(b) a minimum?