Section 4.9

15 $g(t) = \frac{1+t+t^2}{\sqrt{t}} = \frac{1}{\sqrt{t}} + \frac{t}{\sqrt{t}} + \frac{t^2}{\sqrt{t}}$

$= t^{-1/2} + t^{1-\frac{1}{2}} + t^{2-\frac{1}{2}}$

$= t^{-1/2} + t^{1/2} + t^{3/2}$

$\to G(t) = \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{t^{1/2+1}}{\frac{1}{2}+1} + \frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$

$= \frac{t^{1/2}}{1/2} + \frac{t^{3/2}}{3/2} + \frac{t^{5/2}}{5/2} + c$

$= 2t^{1/2} + \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + c$

$=$

18 $g(v) = 2\cos v - \frac{3}{\sqrt{1-v^2}}$

$= 2(\cos v) - 3\left(\frac{1}{\sqrt{1-v^2}}\right)$

$\Rightarrow G(v) = 2\sin v - 3\sin^{-1} v + c$

$$\Rightarrow \quad G(v) = 2\sin v - 3\sin v + C$$

(33) $f'(t) = \dfrac{4}{1+t^2}, \quad f(1) = 0$

$$= 4\left(\frac{1}{1+t^2}\right)$$

$$\Rightarrow \quad f(t) = 4\tan^{-1} t + C$$

But

$$f(1) = 0 \Rightarrow 4\tan^{-1}(1) + C = 0$$

$$\Rightarrow 4\left(\frac{\pi}{4}\right) + C = 0$$

$$\Rightarrow \pi + C = 0$$

$$\Rightarrow C = -\pi$$

Thus,

$$f(t) = 4\tan^{-1} t - \pi$$

(41) $f''(\theta) = \sin\theta + \cos\theta, \quad f(0) = 3, \quad f'(0) = 4$

$$= -(-\sin\theta) + \cos\theta$$

$$\Rightarrow \quad f'(\theta) = -\cos\theta + \sin\theta + C$$

$$f'(0) = 4 \Rightarrow -\cos 0 + \sin 0 + C = 4$$

$$f'(0) = 4 \Rightarrow -\cos 0 + \sin 0 + c = 4$$

$$\Rightarrow -1 + 0 + c = 4$$

$$\Rightarrow c = 5$$

i.e.,

$$f'(\theta) = -\cos\theta + \sin\theta + 5$$

$$= -(\cos\theta) - (-\sin\theta) + 5\theta^{0}$$

$$\Rightarrow f(\theta) = -\sin\theta - \cos\theta + \frac{5\theta^{0+1}}{0+1} + c$$

$$= -\sin\theta - \cos\theta + 5\theta + c$$

But

$$f(0) = 3 \Rightarrow -\sin 0 - \cos 0 + 5(0) + c = 3$$

$$\Rightarrow -1 + c = 3 \Rightarrow c = 4$$

Hence,

$$f(\theta) = -\sin\theta - \cos\theta + 5\theta + 4$$

45 $f''(x) = e^{x} - 2\sin x, \; f(0) = 3, \; f(\pi/2) = 0$

$$= e^{x} + 2(-\sin x)$$

$$\Rightarrow f'(x) = e^{x} + 2\cos x + c$$

$$\Rightarrow f'(x) = e^x + 2\cos x + c$$

$$= e^x + 2(\cos x) + cx^0$$

$$\Rightarrow f(x) = e^x + 2\sin x + cx + D$$

But

$$f(0) = 3 \Rightarrow e^0 + 2\sin 0 + c(0) + D = 3$$

$$\Rightarrow 1 + D = 3 \Rightarrow D = 2$$

So

$$f(x) = e^x + 2\sin x + cx + 2$$

Also,

$$f(\pi/2) = 0 \Rightarrow e^{\pi/2} + 2\sin(\tfrac{\pi}{2}) + c(\pi/2) + 2 = 0$$

$$\Rightarrow e^{\pi/2} + 2(1) + \frac{\pi}{2}c + 2 = 0$$

$$\Rightarrow e^{\pi/2} + 4 + \frac{\pi}{2}c = 0$$

$$\Rightarrow \frac{\pi}{2}c = -4 - e^{\pi/2}$$

$$\Rightarrow c = -\frac{2}{\pi}\left(4 + e^{\pi/2}\right)$$

Hence

Hence

$$f(x) = e^x + 2\sin x - \frac{2}{\pi}\left(4 + e^{\pi/2}\right)x + 2$$

60 $v(t) = t^2 - 3\sqrt{t}$, $s(4) = 8$

$$= t^2 - 3\left(t^{1/2}\right)$$

$$\Rightarrow s(t) = \int v(t) = \frac{t^{2+1}}{2+1} - 3\left(\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) + c$$

$$= \frac{t^3}{3} - 3\left(\frac{t^{3/2}}{3/2}\right) + c$$

$$= \frac{1}{3}t^3 - 3\left(\frac{2}{3}t^{3/2}\right) + c$$

$$s(t) = \frac{1}{3}t^3 - 2t^{3/2} + c$$

But

$$s(4) = 8 \Rightarrow \frac{1}{3}(4^3) - 2\left(4^{3/2}\right) + c = 8$$

$$\Rightarrow \frac{64}{3} - 2\left(\sqrt{4}\right)^3 + c = 8$$

$$\Rightarrow \frac{64}{3} - 2(8) + c = 8$$

$$\Rightarrow \frac{64}{3} - 16 + c = 8$$

$$\Rightarrow c = 8 + 16 - \frac{64}{3}$$

$$= \frac{72 - 64}{3}$$

$$= \frac{8}{3}$$

Hence, the position of the particle at time t is

$$s(t) = \frac{1}{3}t^3 - 2t^{3/2} + \frac{8}{3}$$

63 $a(t) = 10\sin t + 3\cos t$, $s(0) = 0$, $s(2\pi) = 12$

$$= -10(-\sin t) + 3(\cos t)$$

$$\Rightarrow v(t) = A(t) = -10\cos t + 3\sin t + c$$

i.e.;

$$v(t) = -10\cos t + 3\sin t + c$$

$$= -10(\cos t) - 3(-\sin t) + ct^0$$

$$\Rightarrow s(t) = V(t) = -10\sin t - 3\cos t + ct + D$$

$$\Rightarrow \quad s(t) =$$

i.e.; $$s(t) = -10\sin t - 3\cos t + ct + D$$

But

$$s(0) = 0 \Rightarrow -10\sin 0 - 3\cos 0 + c(0) + D = 0$$

$$\Rightarrow \quad -3(1) + D = 0 \Rightarrow D = 3$$

So

$$s(t) = -10\sin t - 3\cos t + ct + 3$$

Also,

$$s(2\pi) = 12 \Rightarrow -10\sin(2\pi) - 3\cos(2\pi) + c(2\pi) + 3 = 12$$

$$\Rightarrow -10(0) - 3(1) + 2\pi c + 3 = 12$$

$$\Rightarrow \quad 2\pi c = 12 \Rightarrow c = \frac{6}{\pi}$$

Hence, the position of the particle at time t is

$$s(t) = -10\sin t - 3\cos t + \frac{6}{\pi}t + 3$$