**Lecture Note 27** (Ref. text book page 350) 11/02/2021

## 4.9 Antiderivatives

**Definition** A function $F$ is called an **antiderivative** of $f$ on an interval $I$ if $F^{'}(x) = f(x)$ for all $x$ in $I$.

**Theorem** If $F$ is an antiderivative of $f$ on an interval $I$, then the most general antiderivative of $f$ on $I$ is

$$F(x) + C$$

where $C$ is an arbitrary constant.

| Function | Particular antiderivative | | Function | Particular antiderivative |
|---|---|---|---|---|
| $cf(x)$ | $cF(x)$ | | $\sin x$ | $-\cos x$ |
| $f(x)+g(x)$ | $F(x)+G(x)$ | | $\cos x$ | $\sin x$ |
| $x^n \ (n \neq -1)$ | $\dfrac{x^{n+1}}{n+1}$ | | $\sec^2 x$ | $\tan x$ |
| $\dfrac{1}{x}$ | $\ln\lvert x\rvert$ | | $\sec x \tan x$ | $\sec x$ |
| $e^x$ | $e^x$ | | $\dfrac{1}{\sqrt{1-x^2}}$ | $\sin^{-1} x$ |
| $b^x$ | $\dfrac{b^x}{\ln b}$ | | $\dfrac{1}{1+x^2}$ | $\tan^{-1} x$ |

In the table we list some particular antiderivatives. Each formula in the table is true because the derivative of the function in the right column appears in the left column. In particular, the first formula says that the antiderivative

**Example 1** Find the most general antiderivative of the function. (Check your answer by differentiation.)

(a) $f(t) = \dfrac{3t^4 - t^3 + 6t^2}{t^4}$ (b) $r(\theta) = \sec\theta\tan\theta - 2e^\theta$ (c) $f(x) = \dfrac{2x^2+5}{x^2+1}$

**Example 2** Find $f$ where $f^{'}(x) = (x+1)/\sqrt{x}$, $f(1) = 5$.