

Lecture Note 2* (Ref. text book page 366)

5.1 Areas and Distances

Definition Let f be a nonnegative continuous function on $[a, b]$. Then, the area of the region under the graph of f is

$$A = \lim_{n \rightarrow \infty} [f(x_1) + f(x_2) + \cdots + f(x_n)]\Delta x$$

where x_1, x_2, \dots, x_n are arbitrary points in the n subintervals of $[a, b]$ of equal width $\Delta x = (b - a)/n$.

Example 1

- (a) Find an approximation of the area of the region R under the graph of the function $f = 1/x$ on the interval $[1, 2]$. Use $n = 4$ subintervals. Choose the left and right representative points and compare the results.
- (b) Find an approximation of the area of the region R under the graph of the function $f(x) = 1 + x^2$ on the interval $[-1, 2]$. Use $n = 3$ subintervals. Choose the representative points to be the right endpoints of the subintervals.

We can also derive similar result for the distance of a vehicle whose velocity information is given.

Example 2 The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

