**Lecture Note 30** (Ref. text book page 392)

11/10/2021

## 5.3 Fundamental Theorem of Calculus

**The Fundamental Theorem of Calculus, Part 1** If $f$ is continuous on $[a, b]$, then the function $g$ defined by

$$g(x) = \int_a^x f(t)dt \qquad a \le x \le b$$

is continuous on $[a, b]$ and differentiable on $(a, b)$, and $g'(x) = f(x)$.

**Example 1** Find the derivative of following functions.

(a) $h(u) = \int_0^u \frac{\sqrt{t}}{t+1}dt$

(b) $h(x) = \int_1^{e^x} \ln t dt$

**The Fundamental Theorem of Calculus, Part 2** If $f$ is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F$ is any antiderivative of $f$, that is, a function such that $F' = f$.

**Example** 2 Evaluate the integral.

(a) $\int_0^1 (u+2)(u-3)du$

(b) $\int_0^3 (2\sin x - e^x)dx$

**Example 3** What is wrong with the equation? $\int_{-2}^{1} x^{-4}dx = \left.\frac{x^{-3}}{-3}\right|_{-2}^{1} = -\frac{3}{8}$