Lecture Note 31 (Ref. text book page 402) 11/12/2021

5.4 Indefinite Integrals and the Net Change Theorem

Because of the relation between antiderivatives and integrals given by the Fundamental Theorem, the notation $\int f(x)dx$ is traditionally used for an antiderivative of f and is called an **indefinite integral.** Thus

$$\int f(x)dx = F(x) \quad \text{means} \quad F^{'}(x) = f(x)$$

You should distinguish carefully between definite and indefinite integrals. A definite integral $\int_a^b f(x)dx$ is a number, whereas an indefinite integral $\int f(x)dx$ is a function (or family of functions).

Table of Indefinite Integrals

- $\int cf(x)dx = c\int f(x)dx$
- $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
- $\int kdx = kx + C$
- $\int \frac{1}{x}dx = \ln|x| + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
- $\int b^x dx = \frac{b^x}{\ln b} + C$
- $\int e^x dx = e^x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \csc^2 x dx = -\cot x + C$
- $\int \sec^2 x dx = \tan x + C$
- $\int \csc x \cot x dx = -\csc x + C$
- $\int \sec x \tan x dx = \sec x + C$
- $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$
- $\int \frac{1}{x^2+1}dx = \tan^{-1} x + C$

Example 1 Find the general indefinite integral.

(a) $\int \sqrt[4]{x^5}dx$ (b) $\int \left(x^2 + 1 + \frac{1}{x^2+1}\right)dx$ (c) $\int \sec t(\sec t + \tan t)dt$ (d) $\int \frac{\sin 2x}{\sin x}dx$

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x)dx = F(b) - F(a)$$

This principle can be applied to all of the rates of change in the natural and social sciences that we discussed in Section 3.7.

Example 2 Evaluate the integral.

(a) $\int_1^2 \left(\frac{x}{2} - \frac{2}{x}\right) dx$ (b) $\int_1^8 \frac{2+t}{\sqrt[3]{t^2}}dt$ (c) $\int_0^1 (x^{10} + 10^x)dx$ (d) $\int_0^{\pi/4} \sec\theta \tan\theta d\theta$