Name: KEY **Auburn ID:** .... **Section:** 130

1. Let $f(x) = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2 = (x^{1/2} + x^{-1/3})^2 \Rightarrow f'(x) = 2(x^{1/2} + x^{-1/3})(\frac{1}{2}x^{-1/2} - \frac{1}{3}x^{-4/3})$

   (a) (1 pts) Find $f(1)$

   $= \left(\sqrt{1} + \frac{1}{\sqrt[3]{1}}\right)^2 = (1+1)^2 = \boxed{4}$

   (b) (1 pts) Find the equation of tangent line to the graph of f at $x = 1$

   $y - y_1 = m(x - x_1)$

   $y - 4 = m(x-1)$

   $y = \frac{2}{3}(x-1) + 4$

   $= \boxed{\frac{2}{3}x + \frac{10}{3}}$

   $m = f'(x)\big|_{x=1} = 2(1^{1/2} + 1^{-1/3})(\frac{1}{2}(1)^{-1/2} - \frac{1}{3}(1)^{-4/3})$

   $= 2(1+1)(\frac{1}{2} - \frac{1}{3}) = 4(\frac{1}{6}) = \frac{2}{3}$

   (c) (1 pts) Find the equation of normal line to the graph of f at $x = 1$

   $m_1 = -\frac{1}{m}$

   $= -\frac{1}{2/3}$

   $= -\frac{3}{2}$

   $y - y_1 = m_1(x - x_1)$

   $y - 4 = -\frac{3}{2}(x-1)$

   $y = -\frac{3}{2}x + \frac{3}{2} + 4$

   $= \boxed{-\frac{3}{2}x + \frac{11}{2}}$

2. (a) (1 pts) Differentiate $g(x) = 4xe^x \csc x$

   Using product rule twice:

   $g'(x) = (4xe^x)'\csc x + 4xe^x(\csc x)'$

   $= (4e^x + 4xe^x)\csc x + 4xe^x(-\cot x \csc x)$

   $= 4e^x[1 + x - x\cot x]\csc x$

   $= \boxed{4e^x(1 + x - x\cot x)\csc x}$

   (b) (1 pts) Find the 91st derivative of $f(x) = \sin x$

   $f'(x) = \cos x$ 1

   $f''(x) = -\sin x$ 2

   $f'''(x) = -\cos x$ 3

   $f^{(4)}(x) = \sin x$ 4

   $\frac{91}{4} = 22 \text{ R } 3$

   $\Rightarrow f^{(91)}(x) = f'''(x)$

   $= \boxed{-\cos x}$

(c) (1 pts) Evaluate $\lim_{x\to 1} \frac{\sin(x-1)}{x^2+x-2}$

$$= \lim_{x\to 1} \frac{\sin(x-1)}{(x-1)(x+2)}$$

$$= \lim_{x\to 1} \frac{\sin(x-1)}{x-1} \cdot \frac{1}{x+2}$$

$$= \lim_{x\to 1} \frac{\sin(x-1)}{x-1} \cdot \lim_{x\to 1} \frac{1}{x+2}$$

$$= \lim_{x-1\to 0} \frac{\sin(x-1)}{x-1} \cdot \frac{1}{1+2}$$

$$= \lim_{y\to 0} \frac{\sin y}{y} \cdot \frac{1}{3}$$

$$= (1)\cdot \frac{1}{3}$$

$$= \boxed{\frac{1}{3}}$$

3. (a) (1 pts) Find the equation of tangent line to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = m(x-3)$$

$$y - 4 = -\frac{3}{4}(x-3)$$

$$y = -\frac{3}{4}x + \frac{9}{4} + 4$$

$$= \boxed{-\frac{3}{4}x + \frac{25}{4}}$$

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25)$$

$$\Rightarrow 2x + 2y\,y' = 0$$

$$\Rightarrow y' = \frac{-2x}{2y} = \frac{-x}{y}$$

$$m = y'(x)\Big|_{x=3}$$

$$= \frac{-3}{y(3)} = \frac{-3}{4}$$

(b) ($1\frac{1}{2}$ pts) Find the derivative of the function $y = \sin^{-1}(3x+1)$

<u>mthd 1:</u> $y = \sin^{-1}(3x+1)$

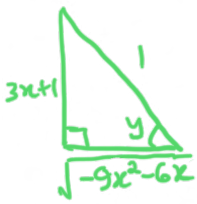
$$\Rightarrow \boxed{\sin y = 3x+1}$$

$$\Rightarrow \frac{d}{dx}(\sin y) = \frac{d}{dx}(3x+1)$$

$$\Rightarrow \cos y \cdot y' = 3$$

$$\Rightarrow y' = \frac{3}{\cos y}$$

$$= \boxed{\frac{3}{\sqrt{-9x^2-6x}}}$$



<u>mthd 2:</u> $y = \sin^{-1}(3x+1)$

$$= \sin^{-1} u, \quad u = 3x+1$$

$$\Rightarrow y'(x) = \frac{1}{\sqrt{1-u^2}} \cdot u'(x)$$

$$= \frac{3}{\sqrt{1-(3x+1)^2}}$$

$$= \boxed{\frac{3}{\sqrt{-9x^2-6x}}}$$

Notice: 2nd mthd requires you to remember that $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(c) ($1\frac{1}{2}$ pts) Use logarithmic differentiation to find the derivative of the function $y = (\sqrt{x})^{3x}$

$$\ln y = \ln(\sqrt{x})^{3x}$$

$$= 3x \ln \sqrt{x}$$

$$= 3x \ln x^{1/2}$$

$$= \frac{3}{2} x \ln x$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(\frac{3}{2}x \ln x\right)$$

$$\Rightarrow \frac{y'}{y} = \frac{3}{2}\ln x + \frac{3}{2}x\left(\frac{1}{x}\right)$$

$$\Rightarrow y' = y\left[\frac{3}{2}\ln x + \frac{3}{2}\right]$$

$$= \boxed{\frac{3}{2}(\sqrt{x})^{3x}(\ln x + 1)}$$