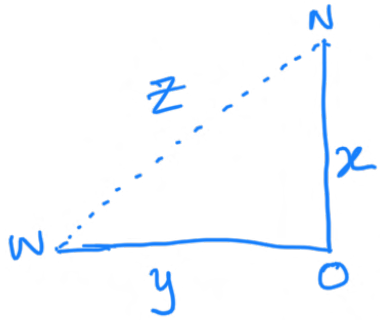
Name: KEY **Auburn ID No.:** .... **Section:** 130

1. ($1\frac{1}{2}$ pts) Two cars start moving from the same point. One travels North at 30 mi/h and the other travels west at 40 mi/h. At what rate is the distance between the cars increasing two hours later?



$\frac{dx}{dt} = 30 \Rightarrow x(t) = 30t$, $\frac{dy}{dt} = 40 \Rightarrow y(t) = 40t$

By Pythagoras rule,

$z^2 = x^2 + y^2 \Rightarrow z = \sqrt{(30t)^2 + (40t)^2} = 50t.$

Hence,

$$\frac{dz}{dt} = 50 \text{ mi/h}$$

2. (a) (1 pts) Find $\lim_{x\to\infty} x\sin(\frac{9\pi}{x})$. Use l'Hospital's Rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to\infty} x \sin\left(\frac{9\pi}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{9\pi}{x}\right)}{1/x} = \lim_{y\to 0} \frac{\sin(9\pi y)}{y} = 9\pi \lim_{y\to 0}\frac{\sin(9\pi y)}{9\pi y} = 9\pi$$

Alternatively:

$$\lim_{x\to\infty} x\sin\left(\frac{9\pi}{x}\right) = \lim_{x\to\infty}\frac{\sin\left(\frac{9\pi}{x}\right)}{1/x} = \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{-\frac{9\pi}{x^2}\cos(9\pi/x)}{-1/x^2} = \lim_{x\to\infty} 9\pi\cos\left(\frac{9\pi}{x}\right) = 9\pi\cos(0) = 9\pi.$$

(b) (1 pts) Use a linear approximation (or differentials) to estimate $(1.99)^5$

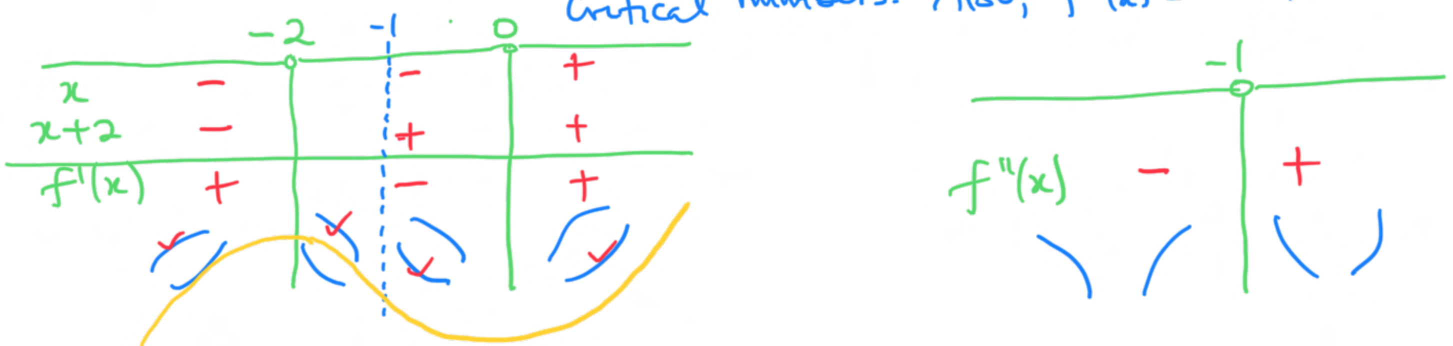
Let $f(x) = x^5$. Since $1.99 \approx 2$,

$L(x) = f(2) + f'(2)(x-2) = 2^5 + 5(2^4)(x-2) = 32 + 80(x-2)$

So

$$(1.99)^5 \approx L(1.99) = 32 + 80(1.99-2) = 32 + 80(-0.01) = 32 - 0.8 = 31.2$$

3. Let $f(x) = x^3 + 3x^2$. $\Rightarrow f'(x) = 3x^2 + 6x = 3x(x+2) \Rightarrow x = 0, -2$ are critical numbers. Also, $f''(x) = 6x + 6 \Rightarrow x = -1$



	$(-\infty,-2)$	$(-2,-1)$	$(-1,0)$	$(0,\infty)$
x	$-$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$	$+$
$f'(x)$	$+$	$-$	$-$	$+$

	$(-\infty,-1)$	$(-1,\infty)$
$f''(x)$	$-$	$+$

(a) ($\frac{1}{2}$ pts) Find the interval(s) where the function f is increasing

$(-\infty, -2),\ (0, \infty)$

(b) ($\frac{1}{2}$ pts) Find the interval(s) where f is decreasing

$(-2, 0)$

(c) ($\frac{1}{2}$ pts) Find the interval(s) where the graph of f is concave upward

$(-1, \infty)$

(d) ($\frac{1}{2}$ pts) Find the interval(s) where the graph of f is concave downward

$(-\infty, -1)$

(e) ($\frac{1}{2}$ pts) Find the local extrema of f

local max point: $(-2, f(-2))$; local min point: $(0, f(0))$

(f) ($\frac{1}{2}$ pts) Find the inflection points, if any, of f

$(-1, f(-1))$

(g) (3 pts) Evaluate f at some convenient points, find $\lim\limits_{x\to\infty} f(x)$, $\lim\limits_{x\to-\infty} f(x)$ and together with your results in (a) - (f), sketch the graph of f

$f(-2) = (-2)^3 + 3(-2)^2 = 4$, $f(-1) = (-1)^3 + 3(-1)^2 = 2$, $f(0) = 0^3 + 3(0)^2 = 0$

$\lim\limits_{x\to\infty} (x^3 + 3x^2) = \infty$, $\lim\limits_{x\to-\infty} (x^3 + 3x^2) = -\infty$. Also, $f(-3) = 0$ (an intercept)