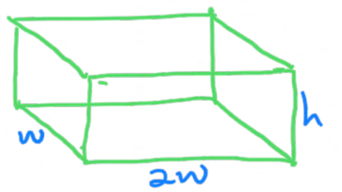
Name: KEY **Auburn ID No.:** .... **Section:** 130

1. (a) (1 pts) A rectangular storage container with an open top is to have a volume of $10m^3$. The length of its base is twice the width. Material for the base costs \$20 per square meter. Material for the sides costs \$12 per square meter. Find the cost of materials for the cheapest such container.

Volume $= lwh = 2w^2h = 10 \Rightarrow h = \frac{5}{w^2}$



The cost function

$C(w) = 20(2w^2) + 12(2(2wh)) + 12(2wh)$

$= 40w^2 + \frac{360}{w}$

So, $C'(w) = 80w - \frac{360}{w^2} \Rightarrow 80w^3 - 360 = 0 \Rightarrow w = \sqrt[3]{\frac{9}{2}}$ is critical number

Also, $C''(w) = 80 + \frac{520}{w^3} > 0$ since $w > 0$. Hence, C attains absolute minimum, $\boxed{C\left(\sqrt[3]{9/2}\right)}$ is the cheapest cost of materials for such container.

(b) ($1\frac{1}{2}$ pts) Find the point on the line $y = 3x + 2$ that is closest to the origin we are required to minimize

$d(x) = \sqrt{x^2+y^2} = \sqrt{x^2+(3x+2)^2} = \sqrt{10x^2+12x+4}$.

Equivalently, we minimize $f(x) = 10x^2+12x+4$.

$f'(x) = 20x + 12 = 0 \Rightarrow x = \frac{-12}{20} = -3/5$.

Also, $f''(x) = 20 > 0 \Rightarrow f$ and consequently d are minimized at $x = -3/5$.

$\Rightarrow y = 3(-3/5) + 2 = \frac{1}{5}$

Hence, the required point on the line is $(x,y) = \boxed{(-3/5, 1/5)}$.

2. (a) ($1\frac{1}{2}$ pts) Find the most general antiderivative of the function $g(x) = \sec x \tan x - 4e^x$

$\int g(x)\,dx = \int (\sec x \tan x - 4e^x)\,dx$

$= \boxed{\sec x - 4e^x + C}$

(b) (2 pts) A particle is moving with acceleration $a(t) = t^2 - 3t + 4$, $s(0) = 0$, $s(1) = 2$. Find the position, $s(t)$ of the particle at time t.

$$v(t) = \int a(t)\,dt = \int (t^2 - 3t + 4)\,dt = \frac{t^3}{3} - \frac{3}{2}t^2 + 4t + C$$

$$s(t) = \int v(t)\,dt = \int \left(\frac{t^3}{3} - \frac{3}{2}t^2 + 4t + c\right)dt = \frac{t^4}{12} - \frac{3}{6}t^3 + 2t^2 + ct + D$$

So, $s(0) = 0 \Rightarrow D = 0$ and $s(1) = 2 \Rightarrow \frac{1}{12} - \frac{3}{6} + 2 + c = 2$

$\Rightarrow c = 5/12$.

Hence,

$$s(t) = \boxed{\frac{t^4}{12} - \frac{1}{2}t^3 + 2t^2 + \frac{5}{12}t}$$

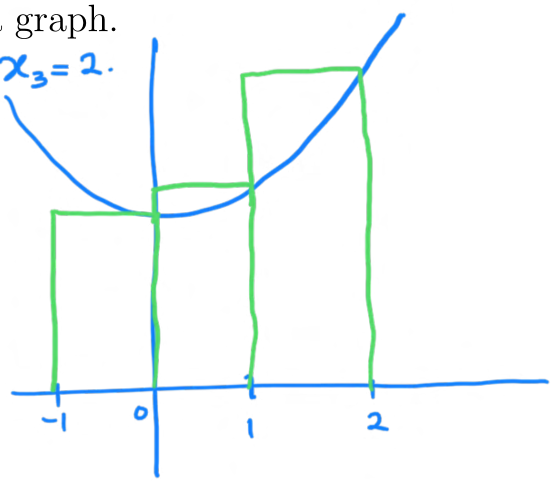
3. (a) ($1\frac{1}{2}$ pts) Let $f(x) = 8 + 2x^2$ from $x = -1$ to $x = 2$.

i. Estimate the area under the graph of f using three rectangles and right endpoints. Illustrate with a graph.

$\Delta x = \frac{2-(-1)}{3} = 1 \Rightarrow x_1 = 0,\ x_2 = 1,\ x_3 = 2.$

So

$$\begin{aligned} \text{Area} &= \Delta x\,[f(x_1) + f(x_2) + f(x_3)] \\ &= 1\,[f(0) + f(1) + f(2)] \\ &= 8 + 10 + 16 \\ &= \boxed{34} \end{aligned}$$



ii. Estimate the area under the graph of f using three rectangles and midpoints $x_1 = -\frac{1}{2}$, $x_2 = \frac{1}{2}$, $x_3 = 3/2$.

So

$$\text{Area} = 1\,[f(-1/2) + f(1/2) + f(3/2)] = 8\tfrac{1}{2} + 8\tfrac{1}{2} + 12\tfrac{1}{2}$$

$$= \boxed{29.5}$$

(b) ($1\frac{1}{2}$ pts) Use the definition to find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x\, f(x_i) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} f\left(-1 + \frac{3i}{n}\right) = \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[8 + 2\left(-1 + \frac{3i}{n}\right)^2\right]}$$

(c) (1 pts) Evaluate $\int_{-1}^{2} (8 + 2x^2)\,dx$

$$= 8x + \frac{2x^3}{3}\Big|_{-1}^{2} = \left[8(2) + \frac{2(2^3)}{3}\right] - \left[8(-1) + \frac{2(-1)^3}{3}\right]$$

$$= \frac{64}{3} + \frac{26}{3} = \frac{90}{3} = \boxed{30}$$

This is the precise area under $f(x) = 8 + 2x^2$ from $x = -1$ to $x = 2$.