TEST-1 REVIEW KEY

1) ⓐ $f(-1) = 3$ ⓑ $x = -2, 1$ ⓒ Domain: $[-2, 6]$
Range: $[-1, 3]$

ⓓ inc: $(-1, 1)$
dec: $(-2, -1) \cup (1, 6)$

2) ⓐ $\log 4 + \log 25 - \log 1 = \boxed{2}$ ⓑ $e^{\ln(\ln 2e)} = \ln 2e = \ln 2 + \ln e = \boxed{\ln 2 + 1}$

ⓒ $\log_2 32 = \boxed{5}$ ⓓ $\ln\left(\frac{1}{e^2}\right) = \ln(e^{-2}) = \boxed{-2}$

3) ⓐ $x = \frac{7 - \ln 6}{4}$ ⓑ $x = \frac{10 + e^2}{3}$

4) $\frac{1}{3} < x < \frac{1 + \ln 2}{3}$

5) $f(2) = 12$, $f(-2) = 16$, $f(a) = 3a^2 - a + 2$, $f(a^2) = 3a^4 - a^2 + 2$,
$2f(a) = 6a^2 - 2a + 4$,
$f(a+h) = 3a^2 + 6ah + 3h^2 - a - h + 2$

$$\frac{f(a+h) - f(a)}{h} = 6a + 3h - 1$$

6) $\frac{f(x+h) - f(x)}{h} = 2x + h - 2 \rightarrow$ difference quotient

7) $\frac{f(x+h) - f(x)}{h} = \frac{-5h}{(x+h+3)(x+3)}$

8) (a) $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

(b) $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)$

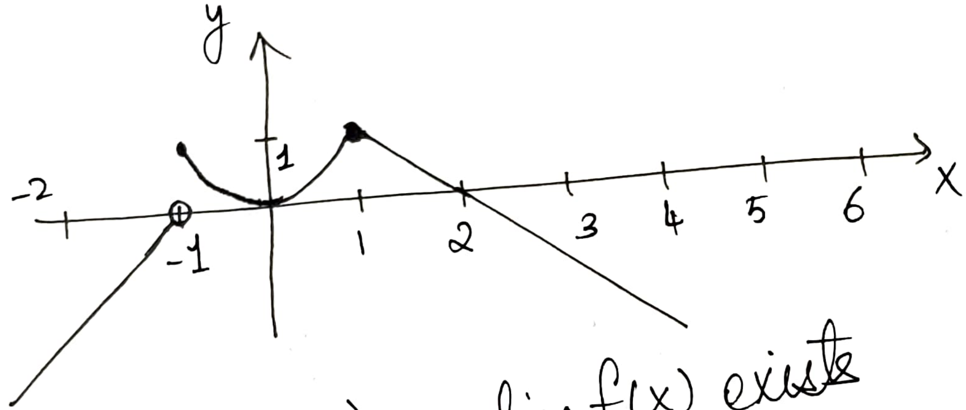
(c) $[-4, 4]$

(d) $(-6, \infty)$

(e) $(-\infty, -1) \cup (-1, \infty)$

9) (a) 2880 bacteria (b) $90 \cdot 2^{t/2}$ bacteria

10)



$(-\infty, -1) \cup (-1, \infty) \to \lim_{x \to a} f(x)$ exists

11) (i) ∞ (ii) DNE (iii) $\frac{1}{3}$ (iv) $-\infty$

(v) DNE (vi) 0 (vii) 0 (viii) $\frac{1}{7}$

(ix) ∞ (x) ∞ (xi) $\frac{6}{5}$ (xii) $-\frac{1}{9}$

(xiii) -10 (xiv) $\frac{2}{3}$

12) Use Intermediate Value theorem:

$f(x) = \ln x + x^2$ is continuous on $[\frac{1}{e}, e]$

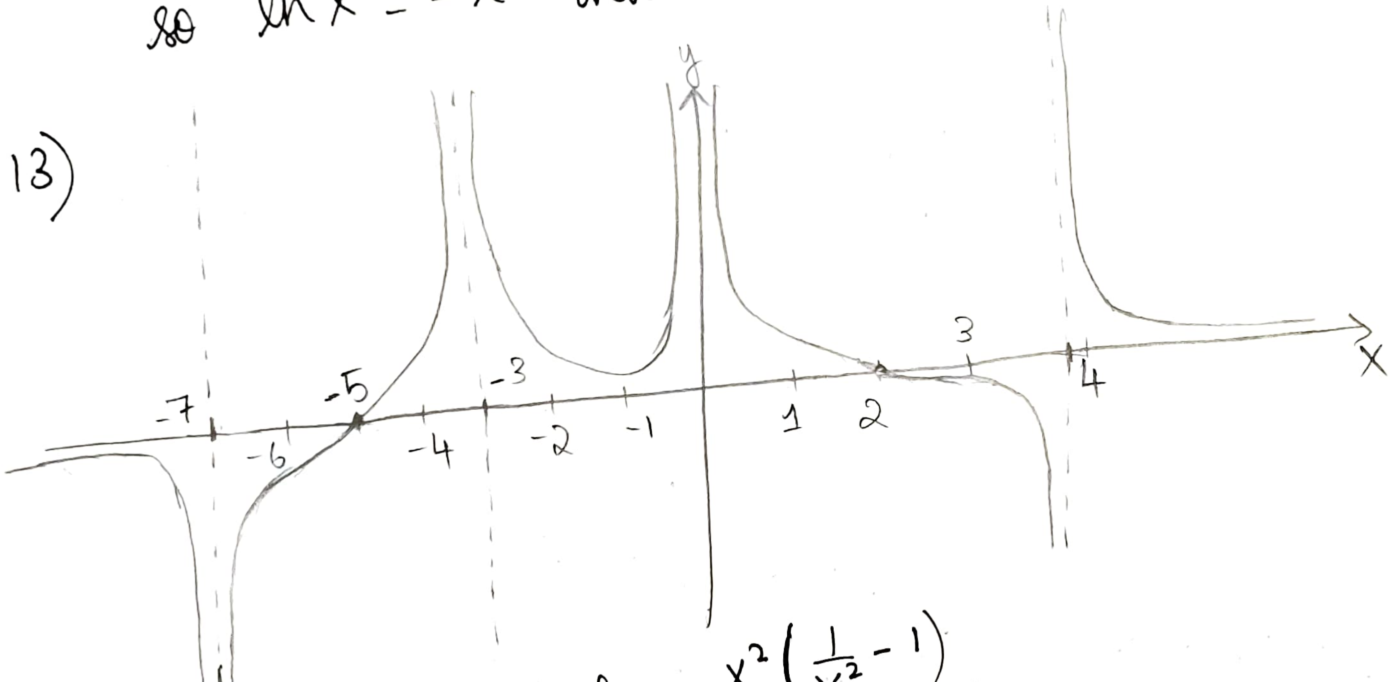
$f\left(\frac{1}{e}\right) = -1 + \frac{1}{e^2} < 0$

$f(e) = 1 + e^2 > 0$

Thus, for some $c \in (1/e, e)$, by IVT, $f(c) = 0$

so $\ln x = -x^2$ has a solution in $\left(\frac{1}{e}, e\right)$.

13)



14) ⓐ $\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} = \lim_{x\to\infty} \frac{x^2\left(\frac{1}{x^2}-1\right)}{x^3\left(1-\frac{1}{x^2}+\frac{1}{x^3}\right)}$

$= \lim_{x\to\infty} \frac{\left(\frac{1}{x^2}-1\right)}{x\left(1-\frac{1}{x^2}+\frac{1}{x^3}\right)}$

$= \lim_{x\to\infty} \frac{1}{x} \cdot \frac{\left(\frac{1}{x^2}-1\right)}{\left(1-\frac{1}{x^2}+\frac{1}{x^3}\right)}$

$= 0 \cdot \left(\frac{0-1}{1-0+0}\right)$

$= \frac{0}{1} = 0$

[since $\lim_{x\to\infty} \frac{1}{x^r} = 0$ for any rational number r]

(b) $\lim_{x\to-\infty} \dfrac{x - x\sqrt{x}}{2x^{3/2}+3x-5} = \lim_{x\to-\infty} \dfrac{x - x^{3/2}}{2x^{3/2}+3x-5}$

$$= \lim_{x\to-\infty} \frac{x^{3/2}\left(\frac{1}{x^{1/2}} - 1\right)}{x^{3/2}\left(2+\frac{3}{x^{1/2}} - \frac{5}{x^{3/2}}\right)}$$

$$= \lim_{x\to-\infty} \frac{\frac{1}{x^{1/2}} - 1}{2+\frac{3}{x^{1/2}} - \frac{5}{x^{3/2}}}$$

[since $\lim_{x\to-\infty} \frac{1}{x^r} = 0$, r is a rational no.]

$$= \frac{0-1}{2+0-0} = \boxed{\frac{-1}{2}}$$

(c) $\lim_{x\to-\infty} \dfrac{x^4-3x^2+x}{x^3-x+2} = \lim_{x\to-\infty} \dfrac{x^4\left(1-\frac{3}{x^2}+\frac{1}{x^3}\right)}{x^3\left(1-\frac{1}{x^2}+\frac{2}{x^3}\right)}$

$$= \lim_{x\to-\infty} \frac{x\left(1-\frac{3}{x^2}+\frac{1}{x^3}\right)}{\left(1-\frac{1}{x^2}+\frac{2}{x^3}\right)}$$

$$= -\infty$$

15) slope $= \dfrac{2x^2-6x+2}{(2x-3)^2}$

$m = \dfrac{2-6+2}{(-1)^2} = -2$

$y - 0 = -2(x-1)$

$\boxed{y = -2x+2}$

16) (a) $v(1) = \lim_{h\to 0} \frac{H(1+h) - H(1)}{h} = 6.28 \text{ m/s}$

(b) $v(a) = \lim_{h\to 0} \frac{H(a+h) - H(a)}{h} = (10 - 3.72\,a) \text{ m/s}$

17) $f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = 3 \rightarrow$ slope of the tangent line at $(1,2)$

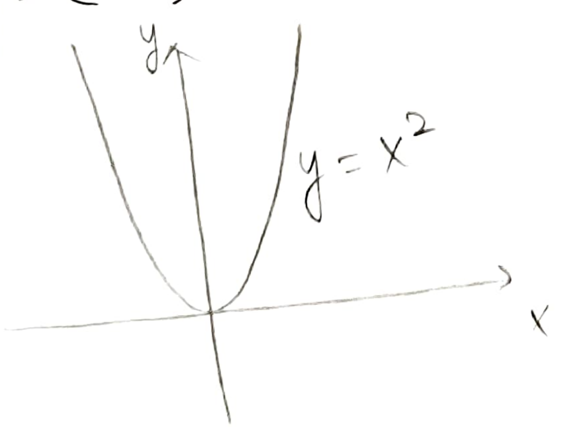
$y - 2 = 3(x-1) \Rightarrow y = 3x - 1$ is the eq^n of the tangent line to the curve at the point $(1,2)$.

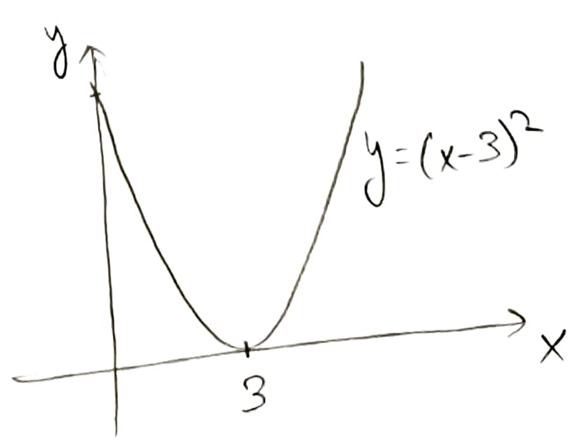
18) $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

(a) $f'(a) = 6a - 4$ (b) $f'(a) = 6a^2 + 1$

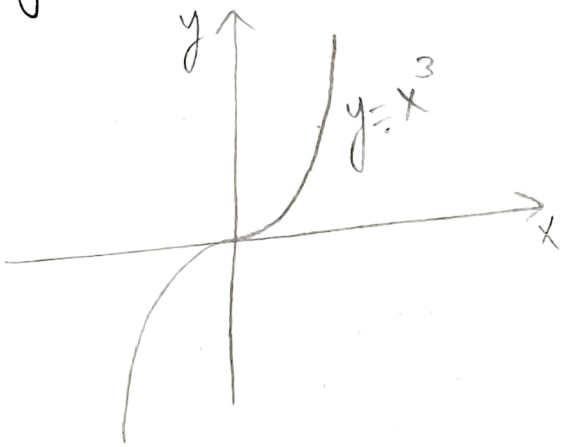
(c) $f'(a) = \frac{5}{(a+3)^2}$ (d) $f'(a) = \frac{-2}{a^3}$

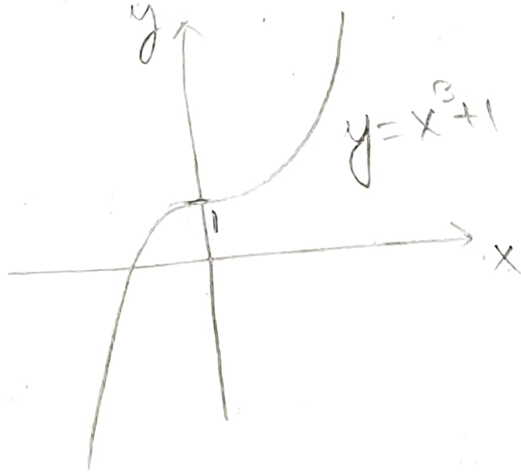
19) (i) $y = (x-3)^2$



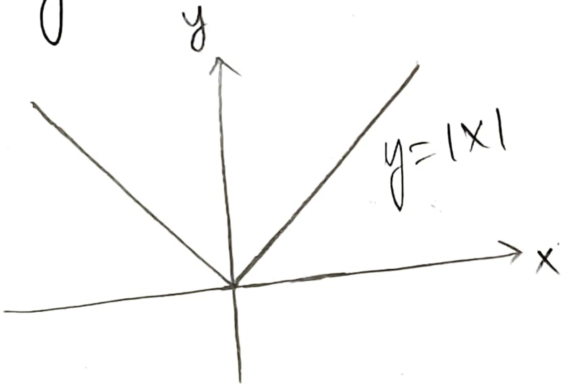


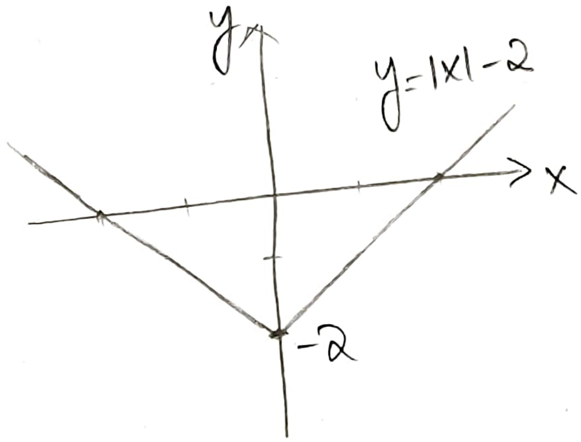
(ii) $y = x^3 + 1$



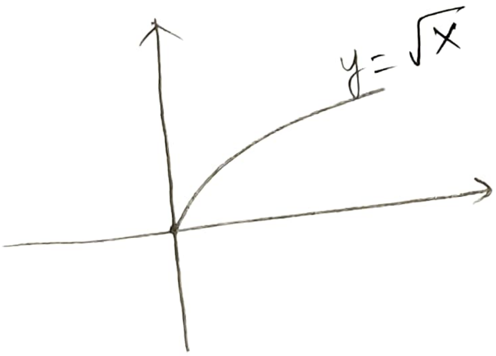


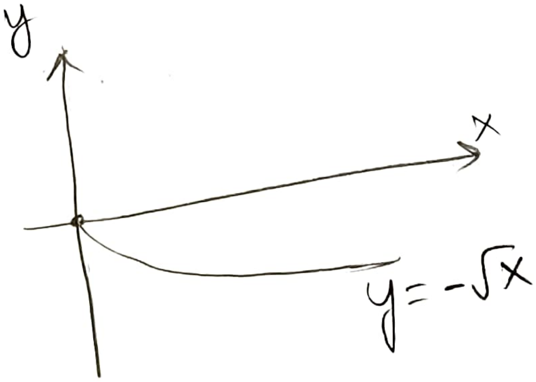
(iii) $y = |x| - 2$

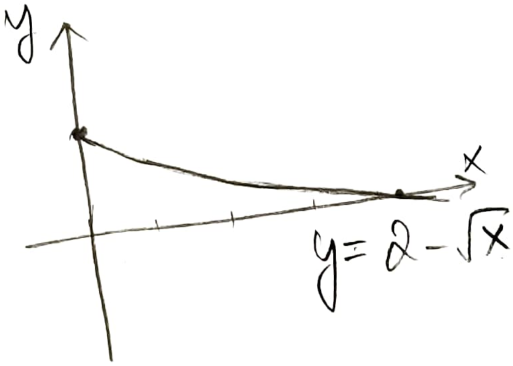




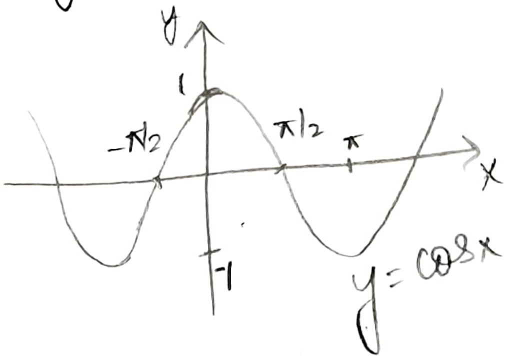
(iv) $y = 2 - \sqrt{x}$

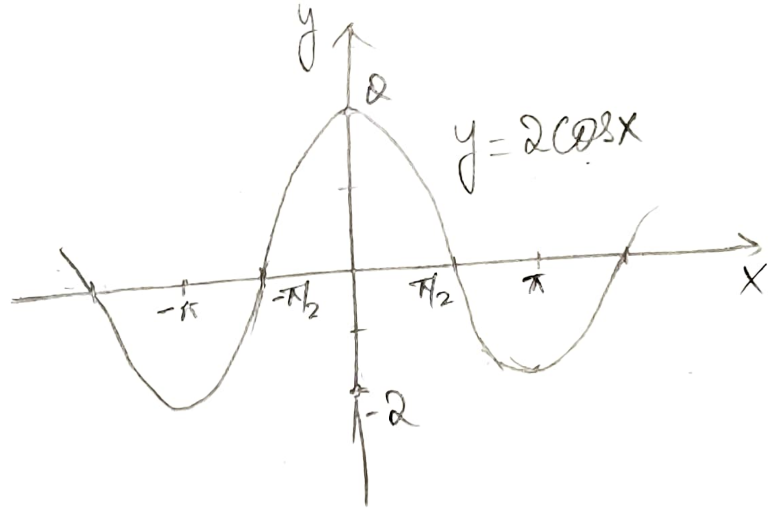


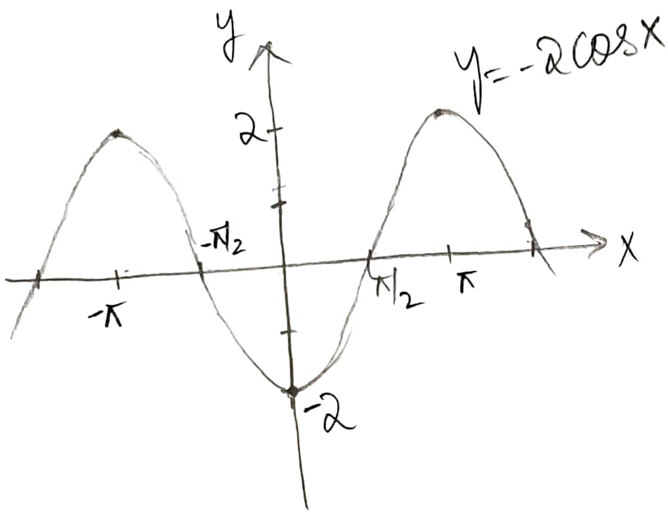


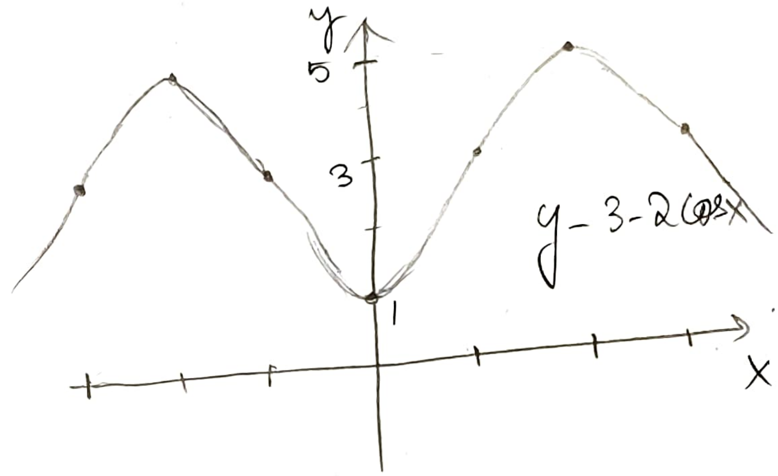


(v) $y = 3 - 2\cos x$

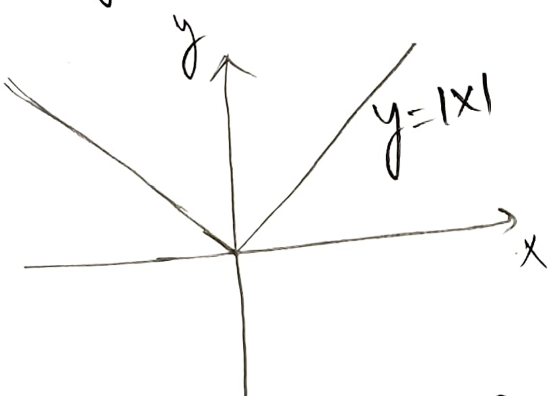


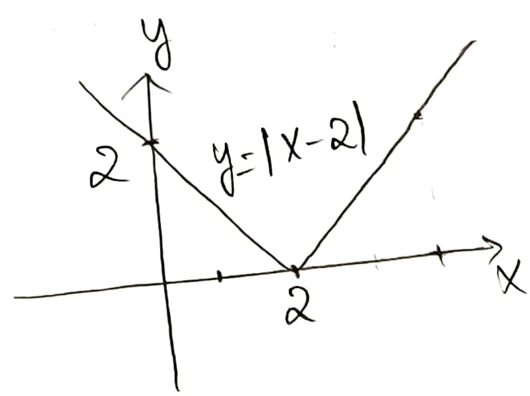






(vi) $y = |x - 2|$





20) (i) $f^{-1}(x) = \dfrac{(x-1)^2 - 2}{3}$

(ii) $f^{-1}(x) = \dfrac{1 + \ln x}{2}$

(iii) $f^{-1}(x) = e^x - 3$

(iv) $f^{-1}(x) = -\ln\left(\dfrac{1-y}{1+y}\right)$

21) (i) $f'(x) = 8 - 10x$

domain of $f(x)$: $(-\infty, \infty)$

domain of $f'(x)$: $(-\infty, \infty)$

(ii) $G'(t) = \dfrac{-7}{(3+t)^2}$

domain of $G(t)$: $(-\infty, -3) \cup (-3, \infty)$

domain of $G'(t)$: $(-\infty, -3) \cup (-3, \infty)$

(iii) $g'(x) = \dfrac{-1}{2\sqrt{9-x}}$

domain of $g(x)$: $(-\infty, 9]$

domain of $g'(x)$: $(-\infty, 9)$