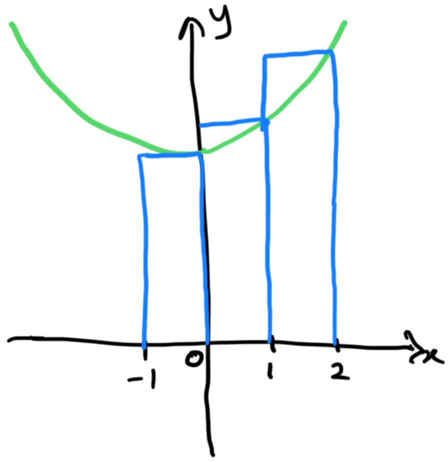
Name: KEY **Auburn ID No.:** .... **Section:** 130

1. (a) (12 pts) Let $f(x) = 5 + x^2$ from $x = -1$ to $x = 2$. Estimate the area under the graph of f using three rectangles and right endpoints. Illustrate with a graph.



$\Delta x = \frac{2-(-1)}{3} = 1; \quad x_1 = -1+1 = 0$

$x_2 = x_1 + 1 = 1, \quad x_3 = x_2 + 1 = 2.$

So

$$\text{Area} \approx \Delta x\left[f(x_1) + f(x_2) + f(x_3)\right]$$
$$= 1\left[f(0) + f(1) + f(2)\right]$$
$$= (5+0^2) + (5+1^2) + (5+2^2)$$
$$= 5 + 6 + 9$$
$$= \boxed{20}$$

(b) (12 pts) Express the integral as a limit of Riemann sums. Do not evaluate the limit. (Use the right endpoints of each subinterval as your sample points.)

$$\int_7^9 \left(x^2 + \sqrt{1+2x}\right)\,dx$$

$\Delta x = \frac{9-7}{n} = \frac{2}{n}; \quad x_i = 7 + i\Delta x = 7 + \frac{2i}{n}$

So,

$$\int_7^9 \left(x^2 + \sqrt{1+2x}\right)dx = \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(7+\frac{2i}{n}\right)^2 + \sqrt{1+2\left(7+\frac{2i}{n}\right)}\right]}$$

2. (a) (13 pts) Find the most general antiderivative of the function. (Check your answer by differentiation. Use C for the constant of the antiderivative.) $f(x) = 7x^{\frac{2}{5}} + 6x^{-\frac{4}{5}} + \sqrt[7]{x^2}$

$$= 7x^{2/5} + 6x^{-4/5} + x^{2/7}$$

So

$$F(x) = \int \left(7x^{2/5} + 6x^{-4/5} + x^{2/7}\right) dx$$

$$= \frac{7x^{\frac{2}{5}+1}}{\frac{2}{5}+1} + \frac{6x^{-\frac{4}{5}+1}}{-\frac{4}{5}+1} + \frac{x^{\frac{2}{7}+1}}{\frac{2}{7}+1} + C$$

$$= \boxed{5x^{7/5} + 30x^{1/5} + \frac{7}{9}x^{9/7} + C}$$

(b) (13 pts) A particle is moving with the given data:

$$a(t) = t^2 + 4t + 3, \quad s(0) = 0, \quad s(1) = 3$$

Find the position of the particle at time, t.

$$v(t) = \int a(t)dt = \int (t^2 + 4t + 3)dt = \frac{t^3}{3} + 2t^2 + 3t + C$$

$$S(t) = \int v(t)dt = \int \left(\frac{t^3}{3} + 2t^2 + 3t + C\right)dt$$

$$= \frac{t^4}{12} + \frac{2}{3}t^3 + \frac{3}{2}t^2 + Ct + D$$

Thus,

$s(0) = D = 0$ and so

$$s(1) = \frac{1}{12} + \frac{2}{3} + \frac{3}{2} + C = 3$$

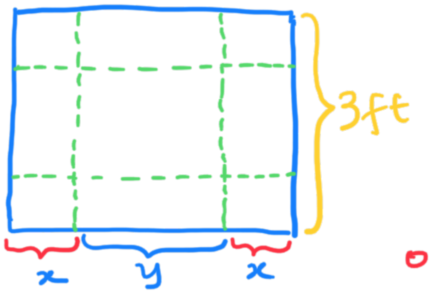
$$\Rightarrow C = 3 - \frac{27}{12} = \frac{9}{12} = \frac{3}{4}$$

Hence, the position of the particle at time, t is

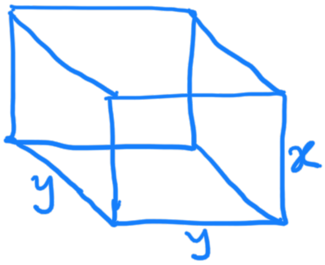
$$S(t) = \boxed{\frac{t^4}{12} + \frac{2}{3}t^3 + \frac{3}{2}t^2 + \frac{3}{4}t}$$

3. A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

(a) (3 pts) Draw a diagram illustrating the general situation. Let x denote the length of the side of the square being cut out. Let y denote the length of the base.



$0 \le x \le 3/2$



(b) (3 pts) Write an expression for the volume V in terms of both x and y.

$$V = lwh = y \cdot y \cdot x = \boxed{xy^2}$$

(c) (3 pts) Use the given information to write an equation that relates the variables x and y.

A side of the cardboard is 3ft. So,

$$x + y + x = 3 \Rightarrow \boxed{y = 3 - 2x}$$

(d) (3 pts) Use part (d) to write the volume as a function of only x.

$$V(x) = xy^2 = x(3-2x)^2 = x(9 - 12x + 4x^2) = \boxed{4x^3 - 12x^2 + 9x}$$

(e) (3 pts) Finish solving the problem by finding the largest volume that such a box can have.

$$V'(x) = 12x^2 - 24x + 9 = 0 \Rightarrow 3(2x-1)(2x-3) = 0 \Rightarrow x = 1/2,\ 3/2$$

$V''(x) = 24x - 24 \Rightarrow V''(1/2) = 24(\frac{1}{2}) - 24 = -12 < 0$ and $V''(3/2) = 24(\frac{3}{2}) - 24 > 0$

Also, $V(0) = 0$, $V(1/2) = 4(\frac{1}{2})^3 - 12(\frac{1}{2})^2 + 9(\frac{1}{2}) = \frac{1}{2} - 3 + \frac{9}{2} = 2$

$V(3/2) = 0$. Hence, the largest possible volume is $\boxed{2 ft^3}$

4. (a) (13 pts) Evaluate the integral.

$$\int_0^1 (x+2)\sqrt{4x+x^2}\,dx$$

Let $u = 4x + x^2$

$\frac{du}{dx} = 4 + 2x$

$\Rightarrow dx = \frac{1}{2(2+x)} du$

Also,

$x = 0 \Rightarrow u = 0$

$x = 1 \Rightarrow u = 4+1 = 5$

$$\int_0^1 (x+2)\sqrt{4x+x^2}\,dx = \int_0^5 (x+2)\, u^{1/2} \cdot \frac{1}{2(2+x)} du$$

$$= \frac{1}{2}\int_0^5 u^{1/2}\, du$$

$$= \frac{1}{3} u^{3/2} \Big|_0^5$$

$$= \frac{1}{3}\left(5^{3/2} - 0^{3/2}\right)$$

$$= \boxed{\frac{1}{3}\sqrt{125}}$$

(b) (12 pts) Find the derivative of the function

$$F(x) = \int_2^{x^2} \left(e^{t^2} + t\right) dt$$

Let $u = x^2$. Then by Continuity of $f(t) = e^{t^2} + t$ and chain rule,

$$F'(x) = \frac{d}{du}\int_2^u (e^{t^2} + t)\,dt \cdot \frac{d}{dx}(x^2)$$

$$= f(u)\, 2x$$

$$= 2x\left(e^{(x^2)^2} + x^2\right)$$

$$= \boxed{2x\left(e^{x^4} + x^2\right)}$$

(c) (10 pts) What is wrong with the equation? (**FTC** is the Fundamental Theorem of Calculus)

$$\int_{-1}^{4} x^{-3}\,dx = \frac{x^{-2}}{-2}\Big|_{-1}^{4} = \frac{15}{32}$$

i. There is nothing wrong with the equation.

ii. $f(x) = x^{-3}$ is not continuous at $x = -1$, so **FTC** cannot be applied

iii. $f(x) = x^{-3}$ is not continuous on the interval $[-1, 4]$ so **FTC** cannot be applied

iv. The lower limit is less than 0, so **FTC** cannot be applied

v. $f(x) = x^{-3}$ is continuous on the interval $[-1, 4]$ so **FTC** cannot be applied

Scratch Work