Questions for recitation 31 March 2021

1. Consider the alternating p-series
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}.$$
For what values of $p \in \mathbb{R}$ is this infinite series divergent, conditionally convergent, or absolutely convergent?

2. Determine if each of the series below converges (absolutely/conditionally) or diverges. If possible, for each convergent series, determine the sum of the series. Be sure to fully motivate your answers.

   (a) $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{\ln(\ln n)}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n!n!}{(2n)!}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+2n}}{(\arctan(n))^n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n^2)3^n n!}{n^n}$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

3. Do the following series converge or diverge? If they converge, is it conditional or absolute?

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1+n}{n^2}\right)$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{\ln(n)}{\ln(n^2)}\right)^{2n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{4/5}}$

   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos(n\pi)}{n^{4/5}}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{5^n}$

4. State a criteria under which
$$\sum_{j=1}^{\infty} \left(\sum_{i=0}^{\infty} ax^i\right)^j$$
converges.