Questions for recitation 26 March 2021

1. Does the following series converge? Why or why not? If it does converge, for what value of m will the mth sum be within .001 of the infinite sum? $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n}$ **Solution:** This converges by the A.S.T., and $|a_n| < .001$ when $\frac{1}{3n} < .001 \implies n > 334$. So we need the 333rd partial sum.

2. Which of the following alternating series converge and which diverge? Be sure to fully support your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{10^n}{n^{10}}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln(n)}$

   (d) $\displaystyle\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln(n)}{\ln(n^2)}$

   (e) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$

   (f) $\displaystyle\sum_{n=1}^{\infty} n^{\frac{-1}{10}} \cos(n\pi)$

   Solution:

   (a) Converges by alternating series theorem.

   (b) Diverges by test for divergence and 10 applications of L'Hospital's rule.

   (c) Converges by alternating series theorem.

   (d) Terms simplify to $(-1)^{n+1}\frac{1}{2} \neq 0$. Series diverges by test for divergence.

   (e) Converges by alternating series theorem.

   (f) Observe that $\cos(n\pi) = (-1)^n$ for integer n. Converges by alternating series theorem.

3. Find an example of a divergent alternating series $\sum a_k$ for which $\lim_{k\to\infty} a_k = 0$.
   Solution: We are missing the requirement for the alternating series test that $|a_n|$ decreasing. So an example that violates this is e.g.

$$a_n = \begin{cases} \frac{1}{n} & n \text{ odd} \\ \frac{-1}{n^2} & n \text{ even} \end{cases}$$