1. Let $f(x) = (\frac{x}{2})^{3/2}$.

   (a) Find the exact length of the curve $y = f(x)$ on $0 \le x \le 32$. Compare this arc length to the length of the line segment connecting $(0, f(0))$ and $(32, f(32))$. Which is longer? Does this answer make sense?

   (b) What is the average value of f over this interval? Must $f(x)$ necessarily equal its average value for some $0 \le x \le 32$? Why or why not?

(a) $y = \left(\frac{x}{2}\right)^{3/2}$ on $0 \le x \le 32.$

$$\frac{dy}{dx} = \frac{3}{2}\cdot\frac{1}{2}\left(\frac{x}{2}\right)^{1/2} = \frac{3}{4}\left(\frac{x}{2}\right)^{1/2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \left(\frac{3}{4}\left(\frac{x}{2}\right)^{1/2}\right)^2 = \frac{9}{16}\left(\frac{x}{2}\right) = \frac{9}{32}x$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{9}{32}x$$

So

$$\text{Arc length} = \int_0^{32} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^{32} \sqrt{1 + \frac{9}{32}x}\, dx$$

Let $u = 1 + \frac{9}{32}x$. Then

$\frac{du}{dx} = \frac{9}{32} \Rightarrow \frac{32}{9}du = dx$

Also, $x = 0 \Rightarrow u = 1 + \frac{9}{32}(0) = 1,$

$x = 32 \Rightarrow u = 1 + \frac{9}{32}(32) = 10$

$$= \int_1^{10} \sqrt{u} \cdot \frac{32}{9} du$$

$$= \frac{32}{9}\int_1^{10} u^{1/2} du$$

$$= \frac{32}{9}\cdot\frac{2}{3}u^{3/2}\Big|_1^{10}$$

$$= \frac{64}{27}\left(10^{3/2} - 1^{3/2}\right)$$

$$= \frac{64}{27}\left(10\sqrt{10} - 1\right) \approx 72.59$$

For the line segment connecting $(0, f(0))$ and $(32, f(32))$:

$$\text{Length} = \sqrt{(f(32) - f(0))^2 + (32-0)^2}$$

$$f(0) = \left(\frac{0}{2}\right)^{3/2} = 0$$

$$f(32) = \left(\frac{32}{2}\right)^{3/2} = (16)^{3/2} = 4^3 = 64$$

$$= \sqrt{(64-0)^2 + (32)^2}$$

$$= \sqrt{64^2 + 32^2}$$

$$= \sqrt{4096 + 1024}$$

$$= \sqrt{5120}$$

$$= 32\sqrt{5} \approx 71.55$$

The arc length is longer than the length of the line segment connecting $(0, f(0))$ and $(32, f(32))$ and this makes sense since $y = \left(\frac{x}{2}\right)^{3/2}$ is not a straight line (and straight line is the shortest path between two distinct points).

(b) $$f_{ave} = \frac{1}{32-0}\int_0^{32}\left(\frac{x}{2}\right)^{3/2}dx$$

$$= \frac{1}{32}\int_0^{32}\frac{x^{3/2}}{\sqrt{8}}\,dx$$

$$= \frac{1}{32\times 2\sqrt{2}}\int_0^{32} x^{3/2}\,dx$$

$$= \frac{1}{64\sqrt{2}}\cdot\frac{2}{5}x^{5/2}\Big|_0^{32}$$

$$= \frac{1}{32\sqrt{2}}\left(\frac{32^{5/2}}{5} - 0^{5/2}\right)$$

$$= \frac{32^{\frac{5}{2}-1}}{5\sqrt{2}}$$

$$= \frac{(2^5)^{3/2}}{5\cdot 2^{1/2}}$$

$$= \frac{2^{\frac{15}{2}-\frac{1}{2}}}{5}$$

$$= \frac{2^7}{5}$$

$$= \frac{128}{5}$$

$$= \underline{\underline{25.6}}$$

Suppose $f(c) = f_{ave} = 25.6$. Then $c = ?$

$$\left(\frac{c}{2}\right)^{3/2} = 25.6 \Rightarrow \frac{c}{2} = 25.6^{\frac{2}{3}}$$

$$\Rightarrow c = 2(25.6)^{2/3}$$

$$\approx 17.3723 \text{ in } [0, 32]$$

So $f(x)$ is equal to its average at $c \approx 17.3723$ in $[0, 32]$.

If you wish to think more rigorously, then notice that f has to be

> **The Mean Value Theorem for Integrals** If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that
>
> $$f(c) = f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx$$
>
> that is, $$\int_a^b f(x)\,dx = f(c)(b-a)$$

continuous on $[a,b]$ in the Mean Value Theorem for Integrals and $y = \left(\frac{x}{2}\right)^{3/2}$ is continuous on $[0, 32]$. So we can conclude from the theorem there is c in $[0, 32]$ such that

$$f_{ave} = f(c)$$

without computing for c.

2. Consider the function $f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$.

(a) Find the exact length of the curve $y = f(x)$ on $-1 \le x \le 1$.

$y = \cosh x = \dfrac{e^x + e^{-x}}{2}$ on $-1 \le x \le 1$.

$\dfrac{dy}{dx} = \dfrac{e^x - e^{-x}}{2} = \sinh x \Rightarrow 1 + \left(\dfrac{dy}{dx}\right)^2 = 1 + \sinh^2 x$

$\Rightarrow \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2} = \sqrt{1 + \sinh^2 x}$

$= \sqrt{\cosh^2 x}$

$= \cosh x$

a

$$\text{Arc Length} = \int_{-1}^{1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-1}^{1} \cosh x\, dx$$

$$= \sinh x \Big|_{-1}^{1}$$

$$= \sinh(1) - \sinh(-1)$$

$$= \sinh(1) + \sinh(1)$$

sinh is odd function.

$$= 2\sinh(1)$$

3. Let $f(x) = \int_1^x \sqrt{t^4 - 1}\, dt$, for $x \ge 1$.

(a) Find the exact length of the curve $y = f(x)$ over $1 \le x \le 3$

$y = \int_1^x \sqrt{t^4-1}\, dt$ on $1 \le x \le 3$.

$$\frac{dy}{dx} = \frac{d}{dx}\int_1^x \sqrt{t^4-1}\, dt = \sqrt{x^4-1}$$

by Fundamental Theorem of Calculus and this is continuous on $[1,3]$.

$$\Rightarrow 1+\left(\frac{dy}{dx}\right)^2 = 1+\left(\sqrt{x^4-1}\right)^2$$

$$= 1+x^4-1$$

$$= x^4$$

$$\Rightarrow \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{x^4} = x^2.$$

So

$$\text{Arc length} = \int_1^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dy$$

$$= \int_1^3 x^2\, dx$$

$$= \frac{x^3}{3}\Big|_1^3$$

$$= \frac{1}{3}\left(3^3-1^3\right)$$

$$= \frac{1}{3}(27-1)$$

$$= \frac{26}{3}$$

4. Find the arc length function $s(x)$ for the curve $y = \frac{(x^2+2)^{3/2}}{3}$, with starting point $(0, \frac{2\sqrt{2}}{3})$

$$y = \frac{(t^2+2)^{3/2}}{3}, \quad 0 \le t \le x$$

$$\frac{dy}{dt} = \frac{3}{2} \cdot 2t \cdot \frac{(t^2+2)^{1/2}}{3} = t\,(t^2+2)^{1/2}$$

$$\Rightarrow 1 + \left(\frac{dy}{dt}\right)^2 = 1 + \left(t\,(t^2+2)^{1/2}\right)^2 = 1 + t^2(t^2+2)$$

$$= 1 + t^4 + 2t^2$$

$$= (1+t^2)^2$$

So,

$$s(x) = \int_0^x \sqrt{1 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^x \sqrt{(1+t^2)^2}\, dt$$

$$= \int_0^x (1+t^2)\, dt$$

$$= t + \frac{t^3}{3} \Big|_0^x$$

$$= \left. t + \frac{t^3}{3} \right|_0^x$$

$$= x + \frac{x^3}{3}$$

5. Find the exact length of the curve $y = \ln(\sec x)$, $0 \le x \le \pi/4$

$$\frac{dy}{dx} = \frac{(\sec x)'}{\sec x} = \frac{\tan x \sec x}{\sec x} = \tan x.$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2 x.$$

So

$$\text{Arc length} = \int_0^{\pi/4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\pi/4} \sec x\, dx$$

$$= \int^{\pi/4} \sec x \left(\frac{\sec x + \tan x}{\ldots} \right) dx$$

$$= \int_0^{\pi/4} \sec x \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right) dx$$

$$= \int_0^{\pi/4} \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$$

$$= \ln\left(\sec x + \tan x\right) \Big|_0^{\pi/4}$$

$$= \ln\left(\sec\left(\tfrac{\pi}{4}\right) + \tan\left(\tfrac{\pi}{4}\right)\right) - \ln\left(\sec(0) + \tan(0)\right)$$

$$= \ln\left(\sqrt{2} + 1\right) - \ln\left(1 + 0\right)$$

$$= \ln\left(\sqrt{2} + 1\right)$$

6. Find the exact length of the curve $y = e^{2x}$, $0 \le x \le 1$

$$\frac{dy}{dx} = 2e^{2x} \implies 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(2e^{2x}\right)^2 = 1 + 4e^{4x}$$

So,

$$\text{Arc length} = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^1 \sqrt{1 + 4e^{4x}}\, dx \qquad u = 1 + 4e^{4x}$$

$4x$

$$= \int_0^1 \sqrt{1+4e^{4x}}\,dx$$

$u = 1 + 4e^{4x}$

$\Rightarrow \frac{du}{dx} = 16e^{4x}$

$\Rightarrow \frac{du}{16e^{4x}} = dx$

$\frac{u-1}{4} = e^{4x} \Rightarrow \ln\left(\frac{u-1}{4}\right) = 4x$

$\Rightarrow \frac{1}{4}\ln\left(\frac{u-1}{4}\right) = x$

$$= \int_0^1 u^{1/2} \cdot \frac{du}{16e^{4x}}$$

$$= \int_0^1 u^{1/2} \cdot \frac{du}{16e^{4\left(\frac{1}{4}\ln\left(\frac{u-1}{4}\right)\right)}}$$

$$= \int_0^1 u^{1/2} \cdot \frac{du}{16e^{\ln\left(\frac{u-1}{4}\right)}}$$

$$= \int_0^1 u^{1/2} \cdot \frac{du}{16\left(\frac{u-1}{4}\right)}$$

$$= \int_0^1 u^{1/2} \cdot \frac{du}{4(u-1)}$$

$$= \frac{1}{4}\int_0^1 \frac{u^{1/2}}{u-1}\,du$$

Could have applied $v = u^{1/2}$ sub here but I thought the method I took would be shorter!

$$= \frac{1}{4}\int_0^1 \frac{\sqrt{u}}{(\sqrt{u}-1)(\sqrt{u}+1)}\,du$$

$$= \frac{1}{4}\int_0^1 \frac{\sqrt{u}-1+1}{(\sqrt{u}-1)(\sqrt{u}+1)}\,du$$

$$= \frac{1}{4}\int_0^1 \frac{1}{\sqrt{u}+1}\,du + \frac{1}{4}\int_0^1 \frac{1}{(\sqrt{u}-1)(\sqrt{u}+1)}\,du$$

$$= \frac{1}{4}\int_0^1 \frac{1+\sqrt{u}-\sqrt{u}}{\sqrt{u}+1}\,du + \frac{1}{4}\int_0^1 \frac{1}{u-1}\,du$$

$$= \frac{1}{4}\int_0^1 1\,du - \frac{1}{4}\int_0^1 \frac{\sqrt{u}}{1+\sqrt{u}}\,du + \frac{1}{4}\int_0^1 \frac{1}{u-1}\,du$$

$v = 1+\sqrt{u}$

$\Rightarrow \frac{dv}{du} = \frac{1}{2}u^{-1/2}$

$\Rightarrow 2\sqrt{u}\,dv = du$

Also $(v-1)^2 = u$

$$= \frac{1}{4}u\Big|_{x=0}^{1} - \frac{1}{4}\int_0^1 \frac{\sqrt{u}}{v}\cdot 2\sqrt{u}\,dv + \frac{1}{4}\ln|u-1|\Big|_{x=0}^{1}$$

$$= \frac{1}{4}\left(1+4e^{4x}\right)\Big|_0^1 - \frac{1}{2}\int_0^1 \frac{(v-1)^2}{v}\,dv + \frac{1}{4}\ln\left(1+4e^{4x}-1\right)\Big|_0^1$$

$$= \frac{1}{4}\left(1+4e^{4}-(1+4)\right) - \frac{1}{2}\int_0^1 \frac{v^2-2v+1}{v}\,dv + \frac{1}{4}\ln\left(4e^{4x}\right)\Big|_0^1$$

$$= \left(e^4-1\right) - \frac{1}{2}\int_0^1 \left(v-2+v^{-1}\right)dv + \frac{1}{4}\left(\ln(4e^4) - \ln(4)\right)$$

$$= \left(e^4-1\right) - \frac{1}{2}\left(\frac{v^2}{2}-2v+\ln v\right)\Big|_{x=0}^{1} + \frac{1}{4}\left(\ln 4 + 4 - \ln 4\right)$$

$$= \left(e^4-1\right) - \frac{1}{2}\left(\frac{(1+\sqrt{u})^2}{2} - 2(1+\sqrt{u}) + \ln(1+\sqrt{u})\right)\Big|_{x=0}^{1} + 1$$

$$= \left(e^4-1\right) - \frac{1}{2}\left(\frac{1+2\sqrt{u}+u}{2} - 2 - 2\sqrt{u} + \ln(1+\sqrt{u})\right)\Big|_{x=0}^{1} + 1$$

$$= (e^4-1) - \frac{1}{2}\left(\frac{1+2\sqrt{u}+u}{2} - 2 - 2\sqrt{u} + \ln(1+\sqrt{u})\right)\Big|_{x=0}^{1} + 1$$

$$= e^4 - \frac{1}{2}\left(\frac{1+2\sqrt{u}+u-4-4\sqrt{u}+2\ln(1+\sqrt{u})}{2}\right)\Big|_{x=0}^{1}$$

$$= e^4 - \frac{1}{4}\left(2\ln(1+\sqrt{1+4e^{4x}}) - 2\sqrt{1+4e^{4x}} + 1 + 4e^{4x} - 3\right)\Big|_0^1$$

$$= e^4 - \frac{1}{4}\left(2\ln(1+\sqrt{1+4e^4}) - 2\sqrt{1+4e^4} - 2 + 4e^4\right) + \frac{1}{4}\left(2\ln(1+\sqrt{5}) - 2\sqrt{5} + 2\right)$$

7. A length of string, chain, etc. which is hanging by both ends is called a catenary. Mathematically, a catenary is described by $f(x) = a\cosh(\frac{x}{a})$, with $a > 0$. What is the length of a catenary with its ends fixed at $(1, 1)$ and $(-1, 1)$? (A bicycle or car with square wheels would drive smoothly along a road lined with inverted catenary speed bumps.)

$y = a\cosh\left(\frac{x}{a}\right)$ on $[-1, 1]$.

$$\frac{dy}{dx} = \sinh\left(\frac{x}{a}\right) \Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2\left(\frac{x}{a}\right)$$

So

$$\text{Length of catenary} = \int_{-1}^{1} \sqrt{1+\sinh^2\left(\frac{x}{a}\right)}\, dx$$

$$= \int_{-1}^{1} \sqrt{\cosh^2\left(\frac{x}{a}\right)}\, dx$$

$$= \int_{-1}^{1} \cosh\left(\frac{x}{a}\right) dx$$

$$= a \sinh\left(\frac{x}{a}\right)\Big|_{-1}^{1}$$

$$= a \sinh\left(\frac{1}{a}\right) - a \sinh\left(\frac{-1}{a}\right)$$

$$= a \sinh\left(\frac{1}{a}\right) + a \sinh\left(\frac{1}{a}\right)$$

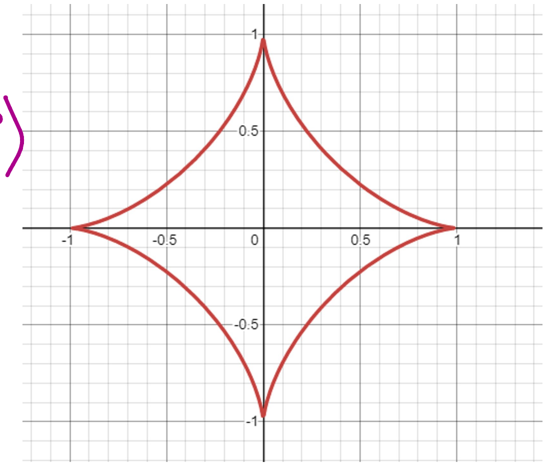
$$= \underline{\underline{2a \sinh\left(\frac{1}{a}\right)}}$$

8. Sketch the astroid given by $x^{2/3} + y^{2/3} = 1$ (there should be a section of curve in each of the 4 quadrants). Find the perimeter of the astroid (hint: use symmetry to reduce the amount of work).

$$x^{2/3} + y^{2/3} = 1 \Rightarrow y^{2/3} = 1 - x^{2/3}$$

$$\Rightarrow y = \left(1 - x^{2/3}\right)^{3/2}$$

(consider the top function only by symmetry)



$$\Rightarrow \frac{dy}{dx} = \frac{3}{2} \cdot -\frac{2}{3} x^{-1/3} \left(1 - x^{2/3}\right)^{1/2}$$

$$= -x^{-1/3}\left(1 - x^{2/3}\right)^{1/2}$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^{-2/3}\left(1 - x^{2/3}\right)$$

$$= 1 + x^{-2/3} - 1$$

$$= x^{-2/3}$$

So, by symmetry (from the graph),

$$\text{Perimeter of astroid} = 4\int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 4 \int_0^1 \sqrt{x^{-2/3}}\, dx$$

$$= 4 \int_0^1 x^{-1/3}\, dx$$

$$= 4 \cdot \frac{3}{2} x^{2/3} \Big|_0^1$$

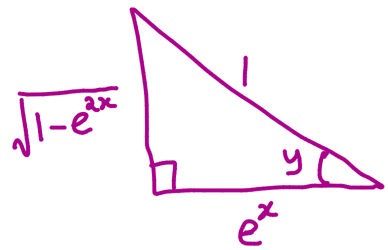
$$= 6\left(1^{2/3} - 0^{2/3}\right)$$

$$= 6$$

9. The function $f(x) = \cos^{-1}(e^x)$ defines a function on $(-\infty, 0]$. Set up 2 integrals (one with respect to x and one with respect to y) to find the arc length of the curve $y = f(x)$ on $-\frac{\ln 2}{2} \leq x \leq 0$. Compute the integral of your choice.

$y = \cos^{-1}(e^x)$ on $\left[-\frac{\ln 2}{2}, 0\right]$.

$\cos y = e^x$:



$$\Rightarrow \cos y = e^x \Rightarrow -\sin y \frac{dy}{dx} = e^x$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x}{-\sin y}$$

$$= \frac{e^x}{-\sqrt{1-e^{2x}}}$$

$$\Rightarrow 1+\left(\frac{dy}{dx}\right)^2 = 1+\left(\frac{e^x}{-\sqrt{1-e^{2x}}}\right)^2$$

$$= 1+\frac{e^{2x}}{(1-e^{2x})}$$

$$= \frac{1-e^{2x}+e^{2x}}{1-e^{2x}}$$

$$= \frac{1}{{}^{2x}}$$

$$= \frac{1}{1-e^{2x}}$$

So,

$$\text{Arc length} = \int_{-\frac{\ln 2}{2}}^{0} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-\frac{\ln 2}{2}}^{0} \sqrt{\frac{1}{1-e^{2x}}}\, dx$$

$$= \int_{-\frac{\ln 2}{2}}^{0} \frac{1}{\sqrt{1-e^{2x}}}\, dx$$

$$u = \sqrt{1-e^{2x}} \quad (\Rightarrow e^{2x} = 1-u^2)$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2}(-2e^{2x})(1-e^{2x})^{-1/2}$$

$$= \frac{-e^{2x}}{\sqrt{1-e^{2x}}}$$

$$\Rightarrow \frac{\sqrt{1-e^{2x}}}{-e^{2x}}\, du = dx$$

Also $x = -\frac{\ln 2}{2}$

$$\Rightarrow u = \sqrt{1-e^{2\left(-\frac{\ln 2}{2}\right)}} = \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

and

$x = 0$

$$\Rightarrow u = \sqrt{1-e^{2(0)}} = 0$$

$$= -\int_{0}^{\frac{1}{\sqrt{2}}} \frac{1}{u} \cdot \frac{\sqrt{1-e^{2x}}}{-e^{2x}}\, du$$

$$= \int_{0}^{\frac{1}{\sqrt{2}}} \frac{1}{u} \cdot \frac{u}{1-u^2}\, du$$

$$= \int_{0}^{\frac{1}{\sqrt{2}}} \frac{1}{(1-u)(1+u)}\, du$$

$$= \int_{0}^{\frac{1}{\sqrt{2}}} \left(\frac{1/2}{1-u} + \frac{1/2}{1+u}\right) du$$

$\frac{1}{\sqrt{2}}$

$$= -\frac{1}{2}\ln|1-u| + \frac{1}{2}\ln(1+u)\Big|_{0}^{\frac{1}{\sqrt{2}}}$$

$$= \left(-\frac{1}{2}\ln\left(1-\frac{1}{\sqrt{2}}\right) + \frac{1}{2}\ln\left(1+\frac{1}{\sqrt{2}}\right)\right) - \left(-\frac{1}{2}\ln 1 + \frac{1}{2}\ln 1\right)$$

$$= -\frac{1}{2}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) + \frac{1}{2}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}} \div \frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

$$= \frac{1}{2}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$

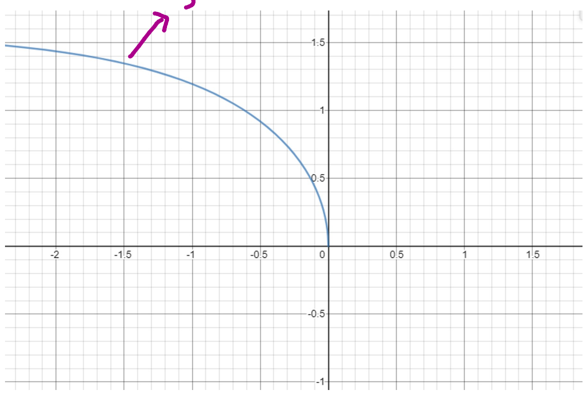
$$= \frac{1}{2}\ln\left(\frac{(\sqrt{2}+1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}\right)$$

$$= \frac{1}{2}\ln\left(\frac{2+2\sqrt{2}+1}{2-1}\right)$$

$$= \frac{1}{2}\ln\left(3 + 2\sqrt{2}\right)$$

Alternatively:

$y = \cos^{-1}(e^x)$



$$y = \cos^{-1}(e^x) \Rightarrow \cos y = e^x \Rightarrow x = \ln(\cos y)$$

and

$$x = -\frac{\ln 2}{2} \Rightarrow y = \cos^{-1}\left(e^{-\frac{\ln 2}{2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4},$$

$$x = 0 \Rightarrow y = \cos^{-1}(e^0) = \cos^{-1}(1) = 0$$

So,

$$\frac{dx}{dy} = \frac{-\sin y}{\cos y} = -\tan y \Rightarrow 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \tan^2 y$$

Thus,

$$\text{Arc length} = \int_0^{\pi/4} \sqrt{1 + \tan^2 y}\; dy$$

$$= \int^{\pi/4} \sqrt{\sec^2 y}\; dy$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 y}\; dy$$

$$= \int_0^{\pi/4} \sec y \, dy$$

$$= \ln\left|\sec y + \tan y\right| \Big|_0^{\pi/4}$$

$$= \ln\left(\sqrt{2}+1\right) - \ln\left(1+0\right)$$

$$= \ln\left(\sqrt{2}+1\right)$$

The two methods yield same result but the second method is easier.