Questions for recitation 3 March 2021

1. Let $f(x) = (\frac{x}{2})^{3/2}$.
   (a) Find the exact length of the curve $y = f(x)$ on $0 \leq x \leq 32$. Compare this arc length to the length of the line segment connecting $(0, f(0))$ and $(32, f(32))$. Which is longer? Does this answer make sense?
   (b) What is the average value of f over this interval? Must $f(x)$ necessarily equal its average value for some $0 \leq x \leq 32$? Why or why not?

2. Consider the function $f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$.
   (a) Find the exact length of the curve $y = f(x)$ on $-1 \leq x \leq 1$.

3. Let $f(x) = \int_1^x \sqrt{t^4 - 1}\, dt$, for $x \geq 1$.
   (a) Find the exact length of the curve $y = f(x)$ over $1 \leq x \leq 3$

4. Find the arc length function $s(x)$ for the curve $y = \frac{(x^2+2)^{3/2}}{3}$, with starting point $(0, \frac{2\sqrt{2}}{3})$

5. Find the exact length of the curve $y = \ln(\sec x)$, $0 \leq x \leq \pi/4$

6. Find the exact length of the curve $y = e^{2x}$, $0 \leq x \leq 1$

7. A length of string, chain, etc. which is hanging by both ends is called a catenary. Mathematically, a catenary is described by $f(x) = a\cosh(\frac{x}{a})$, with $a > 0$. What is the length of a catenary with its ends fixed at $(1, 1)$ and $(-1, 1)$? (A bicycle or car with square wheels would drive smoothly along a road lined with inverted catenary speed bumps.)

8. Sketch the astroid given by $x^{2/3} + y^{2/3} = 1$ (there should be a section of curve in each of the 4 quadrants). Find the perimeter of the astroid (hint: use symmetry to reduce the amount of work).

9. The function $f(x) = \cos^{-1}(e^x)$ defines a function on $(-\infty, 0]$. Set up 2 integrals (one with respect to x and one with respect to y) to find the arc length of the curve $y = f(x)$ on $-\frac{\ln 2}{2} \leq x \leq 0$. Compute the integral of your choice.