Questions for recitation 29 January 2021

1. Do the following integrals converge or diverge? If they converge, evaluate:

   (a)
   $$\int_{-\infty}^{0} \frac{1}{2x-5}\,dx$$

   (b)
   $$\int_{0}^{1} \frac{1}{4y-1}\,dy$$

   (c)
   $$\int_{0}^{1} \frac{\ln x}{\sqrt{x}}\,dx$$

2. Does $\int_0^\pi \frac{\sin x}{\sqrt{\pi - x}}\,dx$ converge or diverge? (Hint: $\sin(\pi - x) = \sin x$.)

3. (a) Evaluate $\int_0^\infty t^2 e^{-3t}\,dt$

   (b) It's very important in probability theory to construct functions that integrate to one. To do so, we often multiply the functions used in probability models by constants to "normalize" them so that they integrate correctly. Find the normalizing constant c such that
   $$1 = \int_0^\infty ct^2 e^{-3t}\,dt.$$

   (c) Generalize your previous two answers for *any* $\lambda > 0$ and non-negative integer n. That is, find $\int_0^\infty t^n e^{-\lambda t}\,dt$, then find c such that
   $$1 = \int_0^\infty ct^n e^{-\lambda t}\,dt.$$

   (This is called the Gamma distribution, and plays a major role in determining how much time elapses before observing a specified event or events. The exponent n controls "how many" events we wait on while the rate constant λ determines "how often" those events occur.)