Questions for recitation 17 February 2021

1. Determine whether the following definite integrals represent an area or a volume. Determine what shape is described (e.g. triangle, sphere, cone). Sketch the planar region or solid and label its dimensions.

   (a) $\int_0^1 3x\,dx$

   (b) $\int_0^{12} \pi(144 - h^2)\,dh$

   (c) $\int_0^{\sqrt{15}} \sqrt{15 - h^2}\,dh$

   (d) $\int_0^7 5(1 - \frac{h}{7})\,dh$

   (e) $\int_0^6 \pi(3 - \frac{y}{2})^2\,dy$

2. Consider the region in the first quadrant bounded by the curves $y = \sqrt{x}$ and $y = \frac{x^2}{8}$.

   (a) Find the volume of the solid with this region as its base and cross-section perpendicular to the x-axis that are squares.

   (b) Find the volume of the solid with this region as its base and cross-section perpendicular to the y-axis that are squares.

   (c) What is the volume of the solid obtained by revolving this region about the y-axis?

3. Challenging problem: A bowl is shaped like a hemisphere with radius 30 cm. A heavy ball of diameter 10 cm is placed in the bottom of the bowl and water is poured in the bowl to a height of $0 < h < 15$ cm. Find the volume of water in the bowl.

4. Example 9 on §6.2: A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30° along a diameter of the cylinder. Find the volume of the wedge.

5. A couple of weeks ago we derived the "normalization constant" for Gamma probability density functions through the following indefinite integral:

$$\int_0^\infty t^n e^{-\lambda t}\,dt = \frac{n!}{\lambda^{n+1}},$$

   for a positive integer n (the "shape parameter") and constant $\lambda > 0$ (the "rate parameter").

   Use this result to find the volume of the solid generated by revolving the region in the first quadrant below the curve $y = x^n e^{-\lambda x}$ about the x-axis.