Questions for recitation 7 April 2021

1. What is wrong with the following computation (for $x > 0$?)

$$2 < (1 + x + x^2 + x^3 + \cdots + x^n + \dots) + (1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \cdots + \frac{1}{x^n} + \dots)$$

$$= \frac{1}{1-x} + \frac{1}{1-\frac{1}{x}} = \frac{1}{1-x} - \frac{x}{1-x} = 1$$

2. For what values of x do the following series converge (i) absolutely, (ii) conditionally? Justify.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^5(x-3)^{2n}}{4^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n\,[\ln(x)]^{n-1}}{x}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{(3n)!n!}$

3. For the following power series, find the interval of convergence and the sum of the series where it is convergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{4^n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} (x-1)^n$

4. Find the radius of convergence and the interval of convergence for the following series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{2}{n}\right)^n x^n$

   (b) $\displaystyle\sum_{n=1}^{\infty} n!(x-\pi)^n$

   (c) $\displaystyle\sum_{n=3}^{\infty} \left(\frac{x^2-1}{2}\right)^n$ (Note: this does not match our usual form of a "power series"; it is not centered properly. For this reason, the notion of "radius of convergence" may not be well defined as the series may lack symmetry).