Questions for recitation 12 March 2021

1. Find a plausible formula for the nth term in the sequence. Note that n should start at 1 in each case.
   (a) 0,1,1,2,3,5,8,13,...
   (b) 0,1,0,1,0,1,...
   (c) 1,0,-1,0,1,0,...
   (d) -3,-2,-1,0,1,...

2. Consider the sequence given by $a_1 = 1$, $a_n = \left(1 - \frac{1}{n^2}\right) a_{n-1}$.
   (a) Write out the first 4 terms of the sequence.

3. A ball is dropped from a height of ten feet and bounces. Each bounce is $\frac{3}{4}$ the height of the bounce before. So, after the ball hits the floor for the first time, the ball rises to a height of $10(\frac{3}{4}) = 7.5$ feet, and after it hits the floor for a second time, it rises to a height of $7.5(\frac{3}{4}) = 10(\frac{3}{4})^2 = 5.625$ feet.
   (a) What height does the ball rise to after it hits the floor for the nth time?
   (b) Find an expression for the total vertical distance the ball has travelled when it hits the ground for the first, second, and third time.
   (c) Find an expression for the total vertical distance the ball has travelled when it hits the ground for the nth time.

4. Write the first five terms of the following sequences.
   (a) $a_1 = 1$, and $a_{n+1} = a_n + \frac{1}{2^n}$
   (b) $a_1 = 2$, and $a_{n+1} = \frac{a_n}{2}(-1)^{n+1}$
   (c) $a_1 = -2$, and $a_{n+1} = \frac{n a_n}{n+1}$
   (d) $a_1 = 2$, $a_2 = -1$, and $a_{n+2} = \frac{a_{n+1}}{a_n}$

5. Consider the sequence $\{a_n\}$ given by $a_n = \frac{4^n}{n!}$.
   (a) Find $\lim_{n\to\infty} a_n$.

6. Determine whether the sequences below converge or diverge. If a sequence converges, find the limit it converges to.
   (a) $a_n = \frac{\sin n}{n}$
   (b) $a_n = \int_1^n \frac{1}{x^p}\, dx$ for $p > 1$
   (c) $a_n = \frac{1}{n}\int_1^n \frac{1}{x}\, dx$