Questions for recitation 17 March 2021

1. Consider the sequence given by $a_1 = 1$, $a_n = \left(1 - \dfrac{1}{n^2}\right) a_{n-1}$.
   (a) Write out the first 4 terms of the sequence.
   (b) Determine whether the sequence $\{a_n\}$ converges. (Challenge: If it does, find $\lim\limits_{n\to\infty} a_n$.)

2. A ball is dropped from a height of ten feet and bounces. Each bounce is $\frac{3}{4}$ the height of the bounce before. So, after the ball hits the floor for the first time, the ball rises to a height of $10(\frac{3}{4}) = 7.5$ feet, and after it hits the floor for a second time, it rises to a height of $7.5(\frac{3}{4}) = 10(\frac{3}{4})^2 = 5.625$ feet.
   (a) What height does the ball rise to after it hits the floor for the nth time?
   (b) Find an expression for the total vertical distance the ball has travelled when it hits the ground for the first, second, and third time.
   (c) Find an expression for the total vertical distance the ball has travelled when it hits the ground for the nth time.

3. Consider the sequence $\{a_n\}$ given by $a_n = \frac{4^n}{n!}$.
   (a) Show that a_n is eventually decreasing. Conclude that $\lim_{n\to\infty} a_n$ exists.
   (b) Find this limit.

4. Rewrite the sum using Sigma (Σ) notation.
   (a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$
   (b) $1 + 2x + 2^2\frac{x^2}{2!} + 2^3\frac{x^3}{3!} + 2^4\frac{x^4}{4!} + \cdots$
   (c) $\frac{1}{1-x}$ provided that $|x| < 1$

5. On New Year's Day, you begin by saving one penny on the first day, two pennies on the second day, three pennies on the third day, and so forth. How many days until you've saved \$200?

6. Consider the sequence $\{a_n\}$ with partial sums $\displaystyle\sum_{n=1}^{N} a_n = S_N$. Suppose the sequence $\{S_N\}$ satisfies $S_N = \dfrac{\sqrt{N+2}-1}{(N+1)^2}$.
   (a) Find $\sum_{n=1}^{\infty} a_n$.
   (b) Evaluate $\lim\limits_{n\to\infty} a_n$.
   (c) Find a closed formula for a_n (leave your answer unsimplified).

7. Evaluate the following sums.

   (a) $\displaystyle\sum_{k=1}^{\infty} \tan^{-1}(k+1) - \tan^{-1}(k)$

   (b) $\displaystyle\sum_{i=1}^{\infty} \ln(i+1) - \ln(i)$

   (c) $\displaystyle\sum_{j=1}^{\infty} \frac{a}{\sqrt{j+1}} - \frac{a}{\sqrt{j}}$

   (d) $\displaystyle\sum_{i=1}^{\infty} \frac{3}{i^2+2i}$

8. Calculate the infinite sum $0.4 + 0.16 + 0.064 + 0.0256 + \cdots$.

9. Find a formula for the nth partial sum of each series. *Hint for (b):* Write as a telescoping series.

   (a) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots + \frac{2}{3^n} + \cdots$

   (b) $\frac{5}{1\cdot 2} + \frac{5}{2\cdot 3} + \frac{5}{3\cdot 4} + \cdots + \frac{5}{n\cdot(n+1)} + \cdots$